Short Questions

1. Suppose that there are two types of consumers: the Rich, who consist 20% of the population, and the Poor, who consist the remaining 80% of the population. Sushi is a "luxury" for the poor but a "necessity" for the rich (luxury and necessity are used here in terms of their formal economic definitions).

Suppose **aggregate** income goes up by exactly 1%. Furthermore, the income of both types of consumers increases, but not necessarily by the same percentage.

a. Is it possible for aggregate (market) demand for sushi to go up by more that 1%. If so, explain how. (You can use a numerical example, but a precise verbal explanation would also be sufficient.)

Yes. Suppose that each Rich person has income I_R and each poor person has income I_P. Total income in the market is $I = 0.2\ I_R + 0.8\ I_P$. Since we know that aggregate income goes up by 1%, either the income of the rich will increase by more than 1% and the income of the poor by less then 1%, or exactly the opposite will happen. Suppose almost all the increase in aggregate income is due to increases in the income of the poor (for whom sushi is a luxury), and the increase in the income of the rich is almost zero. Then clearly the demand for Sushi will go up faster than income, because it a disproportionate fraction of that income will be spent on Sushi.

b. Conversely, is it possible for aggregate (market) demand for sushi to go up by less than 1%. If so, explain how. (Again, you can use a numerical example, but a precise verbal explanation would also be sufficient.)

Yes. Suppose almost all the increase in aggregate income is due to increases in the income of the rich, and the increase in the income of the poor is almost zero. Then the demand for sushi will go up slower than income, because almost all the income and demand change is due to the rich for whom Sushi is a necessity.

2. Consider the statement:

"A linear demand with slope $\beta_1 = -1$ has a lower price elasticity, $e_{P,Q}$, than a linear demand with a slope $\beta_2 = -2$."

Is this statement true or false? Discuss and justify your answer, using a diagram or algebra as you deem appropriate.

This statement is false. The price elasticity of the linear demand depends on the price, and goes from zero to (minus) infinity. Clearly, for the same price the elasticities of the two demand curves will be the same. If we evaluate the elasticities of the two demands at the same quantity, then either could be higher depending on what is the price that corresponds to that quantity level for each of the two demand curves.

3. Consider the statement:

"A constant price elasticity demand curve with price elasticity of demand, $e_{Q,P}$, equal to -1 has a steeper slope than a constant price elasticity demand curve with price elasticity of demand equal to -0.5."

Is this statement true or false? Discuss and justify your answer, using a diagram or algebra as you deem appropriate.

The general formulas for the two (inverse) demand curves are

$$P = \alpha_1 Q^{-1} \qquad \text{and} \qquad P = \alpha_2 Q^{-2}$$

The respective slopes are given by

$$\frac{\partial P}{\partial Q} = -\alpha_1 Q^{-2} \qquad \text{and} \qquad \frac{\partial P}{\partial Q} = -2\alpha_2 Q^{-3}$$

If we were to compute the slope of the two demand curves at a quantity level equal to, say, 1 unit, the first demand curve would be steeper then the second if $\alpha_1 > 2\alpha_2$, and vice versa. If instead we were instead to compare the slope at the same price (say price equal to 1), again the relative steepness of the demand curves would depend on the relative values of α_1 and α_2. Under the first demand curve, the quantity that corresponds to P=1 is $Q = \alpha_1$. Under the second demand curve, the quantity that corresponds to P=1 is $Q = \alpha_2^{0.5}$. Now, plugging these quantities in the expressions for the slope of the inverse demand, we get

$$\frac{\partial P}{\partial Q} = -\alpha_1 Q^{-2} = -\alpha_1 \alpha_2^{-2} = -\frac{1}{\alpha_1}$$

for the first equation and

$$\frac{\partial P}{\partial Q} = -2\alpha_2 Q^{-3} = -2\alpha_2 \alpha_2^{-1.5} = -\frac{2}{\sqrt{\alpha_2}}$$

for the second equation. Clearly, which one is bigger demands on the relative values of α_1 and α_2.

Even the relative slope of two unitary elasticity demand curves cannot be compared. Let the formula for the two unitary elasticity (inverse) demand curves be

$$P = \alpha_1 Q^{-1} \qquad \text{and} \qquad P = \alpha_2 Q^{-1}$$

The slopes for of these two demand curves are

$$\frac{\partial P}{\partial Q} = -\alpha_1 Q^{-2} \qquad \text{and} \qquad \frac{\partial P}{\partial Q} = -\alpha_2 Q^{-2}$$

Clearly, which slope will be steeper depends on whether $\alpha_1 > \alpha_2$ or the reverse.

4. "A linear demand curve is more elastic than a constant elasticity demand curve."

Briefly discuss the validity (or lack thereof) of the above statement.

Since the linear demand curve has an elasticity that goes from zero to minus infinity, while the constant elasticity demand curve has constant elasticity, either can be more elastic than the other depending on which price (or quantity) we evaluate the elasticity.

5. Consider two individuals, A and B who are the only consumers of a product X in a given town. Individual B buys good X in bulk, so he always pays a lower price for it than individual A (who buys it retail). Moreover, when supply conditions change, the bulk and retail prices do not move up and down in lock step.

Suppose the demand function for individual A is

$$X_A = 20 - P_A$$

and that of individual B is

$$X_B = 10 - P_B$$

Can we refer to the demand of the two individuals as the market demand for X in this town ? Why or why not ?

No. It seems that the two consumers pay different prices for the good. Unless we know that the two prices are related in some particular way (i.e., $P_A = 0.5\ P_B$, which would have happened if, say, individual A was getting some senior citizen discount of 50%), there will be no way to describe the combined demand of the two individuals as a function of a single price.

6. What is the income elasticity of demand for a good ?

The income elasticity of demand for a good is the percentage change in the demand for a good if income goes up by one percent.

Problems

1. A market consists of 10 consumers; 4 men and 6 women. A woman's demand for the product is by:

$$Q_w = 100 - 2P$$

for $P \leq 50$. For $P > 50$, $Q_w = 0$. A man's demand for the product is given by:

$$Q_m = 150 - 5P$$

for $P \leq 30$. For $P > 30$, $Q_m = 0$. Using this information, answer the following:

a. Graph a woman's and a man's demand curve with price on the vertical axis and quantity on the horizontal axis. Label all intercepts.

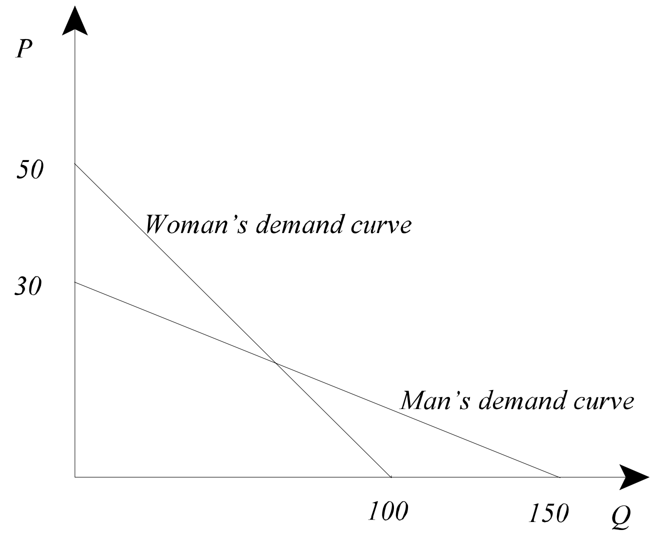
To facilitate drawing the graphs we should first move price on the left hand side of the demand equations. A woman's demand then becomes:

$$P = 50 - \frac{1}{2} Q_w$$

and a man's:

$$P = 30 - \frac{1}{5} Q_m$$

These are plotted below:



A woman's demand curve has a slope of -0.5 and a man's demand curve has a slope of -0.2.

b. How much of the product is demanded by a member of each gender at $P = 35$? At $P = 0$?

At $P = 35$ only women will demand any of the product. Each woman will buy

$$Q_w = 100 - 2 \cdot 35 = 30$$

At $P = 0$, each woman will demand $Q_w = 100$ and each man $Q_m = 150$.

c. What is the market demand for the product at each of the prices specified at part (b)?

The total demand for the product is equal to the sum of the demand of the four men and six women. Since at $P = 35$ only women will demand the product, the total demand is equal to

$$Q_t = 6 \cdot 30 = 180$$

At $P = 0$, the total demand is equal to

$$Q_t = 6 \cdot 100 + 4 \cdot 150 = 1200.$$

d. Use the individual demand curves to derive and graph the total market demand curve for the product with price on the vertical axis and quantity on the horizontal axis. Label all the important points (slopes and intercepts) in the graph.

The total demand for scrod is the sum of the quantities demanded by the ten individuals :

$$Q_t = 6\ Q_w + 4\ Q_m$$

For $P \leq 30$ both of them will buy the product, and the expression above becomes:

$$Q_t = 600 - 12\ P + 600 - 20\ P$$

$$= 1200 - 32\ P$$

Solving this equation for price yields:

$$P = 37.5 - \frac{1}{32}\ Q_t$$

For $P > 30$ and $P \leq 50$ only women will buy the product scrod, and the total demand is:

$$Q_t = 600 - 12\ P$$

Solving this equation for price yields:

$$P = 50 - \frac{1}{12}\ Q_t$$

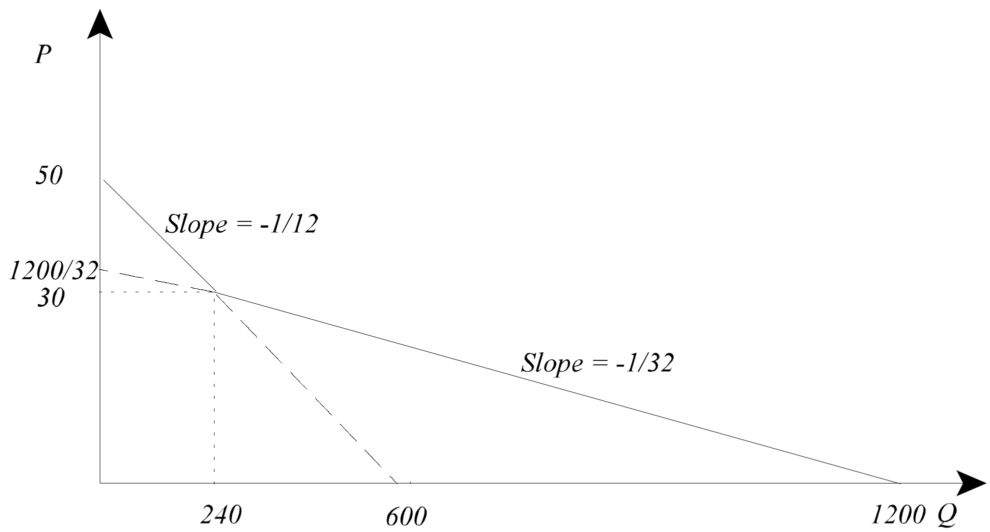
At $P = 30$, when male demand starts kicking in, women are buying $Q_t = 600 - 12 \cdot 30 = 240$ units of the product. For $P > 50$, $Q_t = 0$, as neither group will buy any of the product.

Summarizing the above, we can write the market demand for scrod as:

$$P = 37.5 - \frac{1}{32} Q_t \qquad \text{for } 240 < Q_t < 1200$$

$$P = 50 - \frac{1}{12} Q_t \qquad \text{for } 0 < Q_t < 240$$

The market demand can be graphically depicted as the horizontal addition of the individual demand curves, as shown in the graph below.



2. All consumers' utility functions for gasoline, *G*, and consumption of other goods, *Y*, are given by the utility function $U(G,Y) = G^{\alpha}\ Y^{1-\alpha}$. All consumers have income that equals 2,000. The current price of gasoline is 2 and the price of other goods is normalized to 1.

a. What is the budget constraint of the consumers?

The budget constraint of the consumers is

$$P_G\ G + P_Y\ Y = I$$

which after substituting in for prices and income becomes

$$2\ G + Y = 2000$$

b. Derive the consumers' optimal choice of *G* and *Y*.

The consumers choose *G* and *Y* to maximize utility subject to their budget constraint. Therefore, the optimal choice of *G* and *Y* is obtained from the solution of a constrained maximization problem. The Lagrangian of this problem is

$$\mathcal{L} = G^{\alpha}\ Y^{1-\alpha} + \lambda\ (2000 - 2\ G - Y)$$

The first order conditions of maximization with respect to *G*, *Y*, and the Lagrange multiplier are

$$\frac{\partial \mathcal{L}}{\partial G} = 0 \quad \Rightarrow \quad \alpha\ G^{\alpha-1}\ Y^{1-\alpha} - 2\ \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial Y} = 0 \quad \Rightarrow \quad (1-\alpha)\ G^{\alpha}\ Y^{-\alpha} - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \quad \Rightarrow \quad 2000 - 2\ G - Y = 0$$

The first two equations can be rewritten as

$$\alpha\ G^{\alpha-1}\ Y^{1-\alpha} = 2\ \lambda$$

$$(1-\alpha)\ G^{\alpha}\ Y^{-\alpha} = \lambda$$

Dividing these two equations term by term, we obtain

$$\frac{\alpha\ G^{\alpha-1}\ Y^{1-\alpha}}{(1-\alpha)\ G^{\alpha}\ Y^{-\alpha}} = 2 \quad \Rightarrow$$

$$\frac{\alpha\ Y}{(1-\alpha)\ G} = 2 \quad \Rightarrow$$

$$Y = 2\ \frac{1-\alpha}{\alpha}\ G$$

Substituting this into the budget constraint we obtain

$$2000 - 2\ G - 2\ \frac{1-\alpha}{\alpha}\ G = 0 \quad \Rightarrow$$

$$2\ \alpha\ G + 2\ (1-\alpha)\ G = 2000\ \alpha$$

Solving for G, we obtain the optimal consumption level of gasoline, which is

$$G = 1000\ \alpha$$

per consumer.

c. The government decides to discourage gasoline consumption in order to reduce CO_2 emissions. To do so, it imposes a 20% tax on gasoline, so its price increases to 2.40. All the proceeds from this tax are distributed equally to the consumers in the form of a rebate. Write the budget constraint that the consumers now face. Derive the consumers' optimal choice of G under this tax-plus-rebate environment.

Let G_{tax} be the new, post-tax, level of gasoline consumption per capita. Then, the per capita tax revenue collected by the government is equal to $0.4\ G_{tax}$. This amount is returned to every consumer (regardless of how much they themselves consume). The new post budget constraint of a consumer is

$$2.40\ G + Y = 2000 + 0.40\ G_{tax}$$

where G is his own personal consumption of gasoline.

Setting up the Lagrangian under the new prices and income and taking the first order conditions yields the system

$$\frac{\partial \mathcal{L}}{\partial G} = 0 \quad \Rightarrow \quad \alpha \; G^{\alpha-1} \; Y^{1-\alpha} - 2.40 \; \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial Y} = 0 \quad \Rightarrow \quad (1 - \alpha) \; G^{\alpha} \; Y^{-\alpha} - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \quad \Rightarrow \quad 2000 + 0.40 \; G_{tax} - 2.40 \; G - Y = 0$$

Using similar algebraic manipulations as those performed in part (ii), the first two equations yield

$$Y = 2.40 \; \frac{1 - \alpha}{\alpha} \; G$$

Substituting into the budget constraint, we get

$$2000 + 0.40 \; G_{tax} - 2.40 \; G - 2.40 \; \frac{1 - \alpha}{\alpha} \; G = 0$$

Solving this for *G* would yield the optimal individual consumption of gasoline as a function of the average per person consumption of gasoline. But since all individuals are identical in this model, the average per capita consumption will equal the consumption level of the individual. Thus, the above equation yields.

$$2000 - 2 \; G - 2.40 \; \frac{1 - \alpha}{\alpha} \; G = 0 \quad \Rightarrow$$

$$2000 \; \alpha = 2 \; \alpha \; G + 2.40 \; (1 - \alpha) \; G \quad \Rightarrow$$

$$2000 \; \alpha = (2.40 - 0.4 \; \alpha) \; G \quad \Rightarrow$$

$$G = \frac{2000 \; \alpha}{2.4 - 0.4 \; \alpha}$$

Notice that for all $0 < \alpha < 1$ (which is the relevant range for α), consumption of gasoline has been reduced after the imposition of the tax, despite the fact that the money is returned to the consumers.

d. How would your answer in part (iii) differ if the money collected from the gas tax was spent overseas as foreign aid rather than being rebated to the consumers?

The price of gasoline would still go up to 2.40. However, the income of the consumers would remain equal to 2000, as they would no longer be receiving a rebate. The budget constraint would then be

$$2.40\ G + Y = 2000$$

Except for the budget constraint, the first order conditions of the Lagrangian in part (iii) remain unchanged. Therefore, it will still be true that

$$Y = 2.40\ \frac{1-\alpha}{\alpha}\ G$$

Substituting into the budget constraint, we get

$$2000 - 2.40\ G - 2.40\ \frac{1-\alpha}{\alpha}\ G = 0 \quad \Rightarrow$$

$$2000\ \alpha = 2.40\ \alpha\ G + 2.40\ (1-\alpha)\ G$$

Solving for *G* gives the post tax per capita consumption of gasoline to be

$$G = \frac{2000\ \alpha}{2.4}$$

Notice that the gasoline consumption has been reduced further by the fact that the tax revenue is given away as foreign aid. While the effect of tax in part (iii) is mainly to the substitution effect (as the money is returned to the consumers), the effect of the tax in part (iv) is due to both the substitution and income effects, and thus stronger.

3. A country consists of 1,000 individuals. Half of these have no car: they take public transport wherever they go. This group of consumers consumes no gasoline. The other half of the population has a car and consumes 1 unit of gasoline, *G*, for every 10 dollars they spend on other goods, *Y*. The price of gasoline is 2. The price of other goods is normalized to 1. Every individual in this country has an income of 100.

a. What is the consumption of gasoline, *G*, for the individuals who have a car? What is the total consumption of gasoline in this country?

Since the price of other goods is normalized to 1, *Y* stands both for spending on other goods and for the volume of other goods consumed. The statement that a car-owner consumers 1 unit of gasoline for every 10 dollars spend on other goods is equivalent to the mathematical expression

$$G = \frac{1}{10} Y \tag{1}$$

The consumer's budget constraint is

$$P_G G + Y = 100$$

Plugging in the price of gasoline (which is 2) and using the information from equation (1) above we get

$$2 G + 10 G = 100 \quad \Rightarrow$$

Solving for *G* we obtain the optimal gasoline consumption level for each individual with a car,

$$G = \frac{25}{3} \approx 8.33 .$$

Since there are 500 individuals with a car, the total consumption of gasoline in the market is

$$G_{market} = \frac{12500}{3} \approx 4166.67$$

b. The government wants to discourage driving because it creates too much traffic. For this reason, it imposes a 50% tax on gasoline, the price of which now increases to 3. The government collects the tax revenue and returns it equally to all the consumers as a rebate, whether or not they have a car (that is, each consumer gets the same rebate regardless of how much gasoline he or she consumes). What is the consumption of gasoline under this tax-plus-rebate environment for the individuals who have a car? What is the total consumption of gasoline in this country?

Consumers who have a car still have the same preferences. Therefore, it will still be true that

$$G = \frac{1}{10} Y$$

for each consumer.

Let the new level of per-person consumption of gasoline be G_2. Then, the government is collecting $500\ G_2$ units of revenue from the gasoline tax, and returns $1/2\ G_2$ units of revenue to each consumer (regardless of whether he has or does not have a car). Therefore, the new budget constraint of a consumer is

$$3\ G_2 + Y_2 = 100 + 1/2\ G_2$$

where Y_2 is the new level of spending on other goods. Substituting in the relationship between G and Y for consumers who have a car we get

$$3\ G_2 + 10\ G_2 = 100 + 1/2\ G_2$$

Solving for G_2 we obtain the new per-person consumption level of gasoline

$$G_2 = \frac{100}{12.5} = 8$$

Therefore, the total consumption of gasoline in the country equals

$$G_{market, 2} = 4000.$$

The government was indeed effective in reducing consumption of gasoline (though not by much despite the steep tax!)

c. Which of the two groups (if any) is better off with the tax, excluding any positive effects from the reduction in traffic? Justify your answer with rigorous reasoning or using algebra?

The group without cars is better off with the tax. They have increased income because they get a share of the tax proceeds, but they don't pay higher prices because they don't use any gasoline. Thus, the tax also has a re-distributive effect (in addition to discouraging driving): it shifts money from those with cars to those without cars.