Problems

1. A firm producing hockey sticks has a production function given by:

$$q = 2\sqrt{K\,L}$$

In the short run, the firm's amount of capital equipment is fixed at $K = 100$. The cost of capital is $r = \$2$, and the wage rate for L is $w = \$2$.

a. What is the firm's short-run demand for labor as a function of output produced ?

In the short-run, when capital is fixed at K=100, output is given by the short-run production function:

$$q = 2\sqrt{100\,L}$$
$$= 20\sqrt{L}$$

Since labor is the only input that the firm can vary in the short-run, there is a unique amount of labor that would allow the firm to produce any required level of output. This can be found by simply solving the short run production function for L.

$$q = 20\sqrt{L} \quad \Rightarrow$$

$$q^2 = 400\,L \quad \Rightarrow$$

$$L = \frac{q^2}{400}$$

b. Calculate the firm's short-run total cost curve.

The short-run total cost is given by:

$$SRTC = w\,L + r\,\bar{K}$$

Substituting in for w, r, and $\bar{K}$ we get:

$$SRTC = 2 \ \frac{q^2}{400} + 2 \ \ 100$$

$$= \frac{q^2}{200} + 200$$

c. Calculate the short-run average cost curve.

$$SRAC = \frac{SRTC}{q}$$

$$= \frac{\frac{q^2}{200} + 200}{q}$$

$$= \frac{q}{200} + \frac{200}{q}$$

d. What is the firm's short-run marginal cost function ?

$$SRMC = \frac{d\,SRTC}{d\,q}$$

$$= 2 \ \frac{q}{200}$$

$$= \frac{q}{100}$$

e. Which output level minimizes the short run average cost ? [The answer should be a number.]

The fastest way to find the answer is to note that the marginal cost is equal to the average cost when average cost is at its minimum. Equating SRMC with SRAC we get:

$$SRMC = SRAC \quad \Rightarrow$$

$$\frac{q}{100} = \frac{q}{200} + \frac{200}{q} \quad \Rightarrow$$

$$\frac{2q}{200} = \frac{q}{200} + \frac{200}{q} \quad \Rightarrow$$

$$\frac{q}{200} = \frac{200}{q} \quad \Rightarrow$$

$$q^2 = 200^2 \quad \Rightarrow$$

$$q_{AC_{\min}} = 200$$

f. What would be the firm's profit maximizing choice of output if output price is equal to 5 ? How much labor would the firm hire ?

Profit maximization requires a price taking firm to increase its output until marginal cost is equal to the price. For this firm and for price equal to 5, setting MC=P gives us:

$$\frac{q}{100} = 5$$

which implies that the profit maximizing choice of output is:

$$q^* = 500$$

At this level of output, the firm will hire

$$L = \frac{500^2}{400} = 625$$

units of labor.

2. A firm has production function:

$$Q = K^{0.3} L^{0.5}$$

a. What is the marginal product of labor ? What is the marginal product of capital?

$$MP_L = \frac{\partial Q}{\partial L} = 0.5 \ K^{0.3} \ L^{-0.5}$$

$$MP_K = \frac{\partial Q}{\partial K} = 0.3 \ K^{-0.7} \ L^{0.5}$$

b. If the cost of labor is 2 and the cost of capital is 3, in what proportion should the firm employ capital and labor in order to minimize the cost of producing a given amount of output? [That is, how much capital will the firm hire in terms of the amount of labor hired?]

In order to minimize costs the firm will chose the proportions of capital and labor so as to equate the “bang for the buck” of the marginal units of labor and capital. That is, the ratio of marginal products to input prices will be equal for the two inputs.

$$\frac{MP_L}{w} = \frac{MP_K}{r} \quad \Rightarrow$$

$$\frac{0.5 \ K^{0.3} \ L^{-0.5}}{2} = \frac{0.3 \ K^{-0.7} \ L^{0.5}}{3} \quad \Rightarrow$$

$$0.25 \ K = 0.1 \ L \quad \Rightarrow$$

$$K = \frac{2}{5} L$$

c. How much labor and capital will the firm hire in order to produce output q in the lowest possible cost?

Substituting into the production function the cost minimizing quantity of capital in terms of labor and fixing the output to q, we get:

$$q = \left(\frac{2}{5} L\right)^{0.3} L^{0.5} \quad \Rightarrow$$

$$q = \left(\frac{2}{5}\right)^{0.3} L^{0.8}$$

Solving for labor will yield the demand for labor as a function of output.

$$q = 0.760 \; L^{0.8} \quad \Rightarrow$$

$$L = 0.760^{-1.25} \; q^{1.25} \quad \Rightarrow$$

$$L \approx 1.409 \; q^{1.25}$$

Taking this and plugging it into our expression for K in terms of L that we computed in part (b) we get:

$$K = \frac{2}{5} \; 1.409 \; q^{1.25}$$

$$\approx 0.564 \; q^{1.25}$$

d. What is this firm's cost function ? [i.e. the lowest possible cost of producing output q.]

The cost of production is:

$$C = w \; L + r \; K$$

Therefore,

$$C(q) = 2 \; 1.409 \; q^{1.25} + 3 \; 0.564 \; q^{1.25}$$

$$= 2.818 \; q^{1.25} + 1.692 \; q^{1.25}$$

$$= 4.510 \; q^{1.25}$$

e. What is this firm's marginal cost?

$$MC = \frac{\partial C(q)}{\partial q}$$

$$= 4.510 \;\; 1.25 \;\; q^{0.25}$$

$$= 5.6375 \;\; q^{0.25}$$

f. What is this firm's average cost?

$$AC = \frac{C(q)}{q}$$

$$= 4.510 \;\; q^{0.25}$$

h. How much output will this firm produce if the market price is equal to *P*?

Profit maximization requires the firm to increase its output until price equals marginal cost. Therefore, we have:

$$P = MC \quad \Rightarrow$$

$$P = 5.6375 \;\; q^{0.25} \quad \Rightarrow$$

$$P^4 \approx 1010 \; q \quad \Rightarrow$$

$$q \approx 0.00099 \;\; P^4$$

i. How much labor will this firm hire as a function of the market price ?

Taking our answer for part (h) and substituting it in our answer for the labor demand in part (c) we get:

$$L = 1.409 \; \left(0.00099 \, P^4\right)^{1.25}$$

$$\approx 0.001395 \;\; P^5$$

3. Suppose a firm has a production process that is given by the equation $q = 2G + 3E$ where G is the amount of natural gas used and E is the amount of electricity used. Let the price of electricity be equal to 4 and the price of natural gas 3. The firm is facing a demand given by $P = \alpha - q$.

a. How much electricity and how much natural gas should the firm use to produce 12 units of output at lowest cost?

Given that the MRTS of the linear production function is constant, the cost minimizing choice of inputs will be a corner solution. The firm will choose to use the input that yields the higher marginal product per dollar. For electricity, the ratio of marginal product over its price is 3/4, while for natural gas it is 2/3. Electricity yields the higher bang-for-the-buck and therefore the firm will use only electricity and no natural gas.

The required electricity to produce 12 units of output is given by the equation

$$12 = 3E \Rightarrow E = 4$$

b. In general, how much electricity and how much natural gas should the firm use to produce q units of output at lowest cost?

Following the discussion in part (a), the firm will choose to use only electricity to produce any level of output. The amount of electricity used is given by the equation

$$q = 3E \Rightarrow E = \frac{q}{3}$$

c. What is this firm's cost function?

Since the firm will only use electricity, it's cost function is given by the price of electricity times the amount of needed electricity, or

$$C(q) = 4\frac{q}{3}$$

d. How many units of electricity and how many units of natural gas would the firm demand to produce the profit maximizing level of output? How are they affected by an increase in the demand for the product? (i.e., by an increase in the α?)

The firm's profit maximizing output is obtained by equating marginal revenue with marginal cost, or (given that the firm is facing a linear demand curve) by

$$\frac{4}{3} = \alpha - 2q$$

Therefore, the profit maximizing output is

$$2q = \alpha - \frac{4}{3} \Rightarrow$$

$$q = \frac{\alpha}{2} - \frac{2}{3}$$

This implies that the demand for electricity would be

$$E^* = \frac{\alpha}{6} - \frac{2}{9}$$

An increase in demand increases the demand for electricity.