Short Questions

1. A firm's output level is equal to 10 and the price of its product is set at 20. You are hired by the firm to do a market analysis and recommend any improvements in the firm's pricing strategy. Your market analysis shows that the slope of the demand curve at the current price is equal to -2, that is, you find that dP/dq = -2 when q=10. You inquire about the firm's cost structure and the managers tell you the firm's cost function is $C(q) = 2 + 0.25\ q^2$.

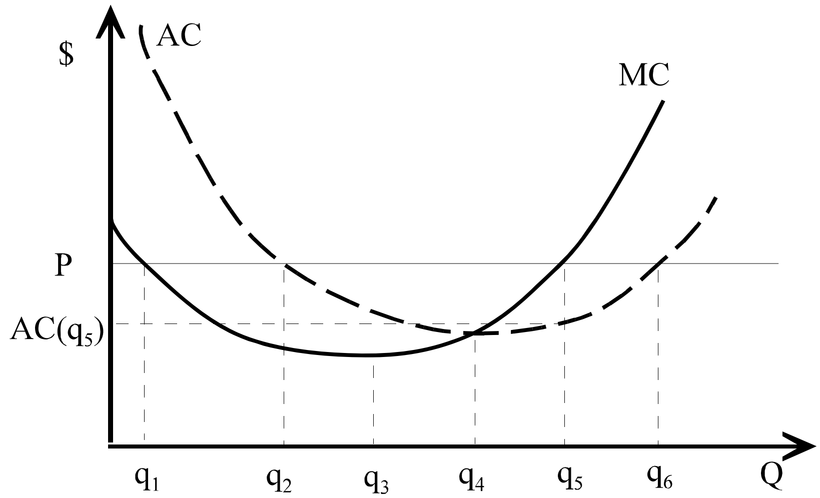
Would you recommend to this firm a price increase, a price decrease, or no price change? Show your reasoning mathematically. Before you proceed with your solution, reflect on the fact that you cannot identify what is the optimal price for this firm; you can only tell whether the firm should increase it (by at least a little bit), decrease it (by at least a little bit), or keep it the same.

The firm should increase output if $MR(q) > MC(q)$, decrease output if $MR(q) < MC(q)$ and keep output the same if $MR(q) = MC(q)$.

The marginal revenue is given by $MR(q) = P'(q)\ q + P(q)$. Using the information above, we can calculate it to be $MR(10) = -2 \cdot 10 + 20 = 0$. Clearly, since marginal cost will be something positive, marginal revenue at the current output level is lower than marginal cost.

Thus, the firm should decrease output, that is, it should increase its price.

2. A perfectly competitive firm's marginal cost and average cost curves are plotted in the figure below, along with the market price.



i. Which output level maximizes the firm's profits?

The profit maximizing output is given by the equality of MC with P that also satisfies the second order conditions (i.e., the intersection between the MC and P lines at which the MC line crosses the price line from below). This output level is q_5. Notice that slightly to the left of q_5 MC < P and slight to the right of q_5 MC > P.

ii. What is the profit per unit output that corresponds to the answer in (i). Draw a line to illustrate your answer, or somehow label it clearly in the above figure.

It is equal to the market price minus the average cost at the optimal output level i.e., it is given by the difference $P - AC(q_5)$ as labeled in the graph.

iii. Which output level maximizes the profit per unit of output produced?

Given that market price is constant and independent of output, the output level that maximizes the profit per unit of output produces is the output level that minimizes the average cost. This is q_4, since the minimum of the average cost is obtained by the intersection of the marginal cost and average cost curves.

iv. Circle in the graph or list below any output level that satisfies the first order conditions of profit maximization.

These are the output levels for which MC=P, i.e., q_1 and q_5.

Problems

1. [Note: This problem cannot be solved using calculus. It must be solved from first principles, i.e., from the construction of the town's revenue and cost functions as a function of the number of employees.] A tourist town with a parking problem is considering how many parking attendants to employ. Each parking attendant costs the town $40,000 per year in salaries, benefits, and work equipment. The parking fine in this town is $20. The probability of catching a car parked illegally is a function of the number of the number of attendants working and given by the table below:

Probability of catching an illegally parked car	Number of attendants employed
0.25	1
0.45	2
0.60	3
0.70	4
0.75	5

a. What would be the cost of employing 1 parking attendant. What would it be for 2 parking attendants ?

One attendant will cost the town 40,000 dollars. Two will cost it 2 40,000 = 80,000 dollars.

b. Suppose that there are two attendants. What is the expected cost of parking illegaly assuming there are no other costs except the fine ?

The expected ticket cost, T, of parking illegally is the probability of getting caught times the fine. Since the probability of getting caught when there are 2 attendants is 0.45, we have,

$$T_2 = 0.45 \ 20 = 9$$

c. What would this cost be if there are 3 attendants ? How about if there are 4 ?

$$T_3 = 0.60 \ 20 = 12$$

$$T_4 = 0.70 \ 20 = 14$$

d. Suppose the number of tourists who are willing to risk parking illegally when the expected ticket cost of parking illegally is T, is given by the equation N=200 (100-T). One can think of this relationship as the "demand for parking" in the city, with the expected ticket cost being the "price" of parking. Plot the demand for parking with the ticket cost on the vertical axis and the quantity (number of tourists parking illegally) on the horizontal axis.

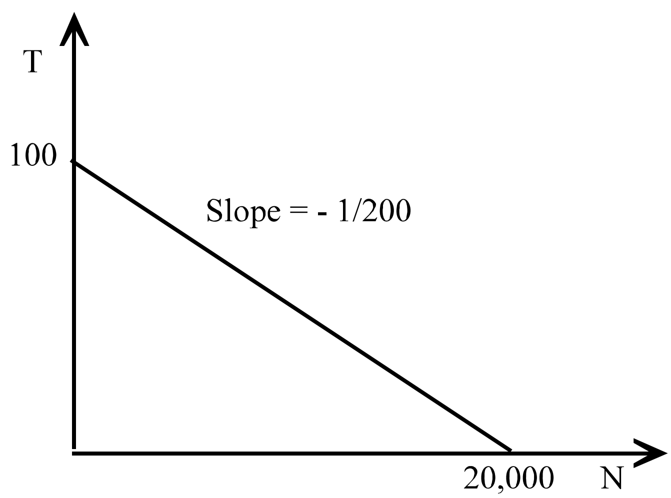
To plot the demand for parking with T on the vertical axis, we must first solve the equation above for T.

$$N = 200\ (100 - T) \Rightarrow$$

$$\frac{N}{200} = 100 - T \Rightarrow$$

$$T = 100 - \frac{N}{200}$$

This yields the linear demand curve shown below:



e. What is the town's expected revenue from parking tickets when there are 2 parking attendants ?

If there are two attendants the cost of parking illegally is 9 dollars. Therefore, the number of tourists who would park illegally would be:

$$N_2 = 200\ (100 - 9) = 200\ 91 = 18{,}200$$

Since the expected revenue per person when there are 2 attendants is 9 dollars, the total expected revenue for the town would be:

$$R_2 = N_2 \; T_2 = 18{,}200 \;\; 9 = 163{,}800$$

f. What is it for 3 attendants ? How about for 4 ?

$$R_3 = N_3 \; T_3$$
$$= 200 \; (100 - 12) \; 12$$
$$= 211{,}200$$

The revenue when there are 3 attendants will be 211,200 dollars. For 4 attendants we have:

$$R_4 = N_4 \; T_4$$
$$= 200 \; (100 - 14) \; 14$$
$$= 240{,}800$$

g. How many attendants should the town hire if it wants to maximize its profit from parking violations ?

The town's profit is the ticket revenue minus the cost of hiring the attendants. The profit with 2 attendants is:

$$\pi_2 = R_2 - C_2 = 163{,}800 - 80{,}000 = 83{,}800$$

With three attendants we have:

$$\pi_3 = R_3 - C_3 = 211{,}200 - 120{,}000 = 91{,}200$$

With four attendants we have:

$$\pi_4 = R_4 - C_4 = 240{,}800 - 160{,}000 = 80{,}800$$

One can check that the profit with 5 attendants is even lower than that with four. Therefore, the profit maximizing number of attendants is equal to 3.

2. Consider a firm that makes talking, mechanical, kangaroo toys. The demand for talking, mechanical kangaroos is given by:

$$P = \alpha I - \frac{\beta}{N} Q$$

where *I* is the per capita income in this market and *N* is the number of kids in this market that are aged 5 to 12.

Suppose the marginal cost of a kangaroo is equal to *MC*. There are no fixed costs.

The firm chooses how many kangaroos to make to maximize its profits.

a. Write the profit function of the firm in terms of the number of kangaroos that it sells. That is, write how much profit the firm would make if it sold *Q* kangaroos.

The profit function of the firm is given by

$$\Pi = P(Q)\, Q - C(Q)$$

Substituting in for the market price and the production costs we can write the profits of the firms as

$$\Pi = \left(\alpha I - \frac{\beta}{N} Q \right) Q - MC\, Q$$

b. What choice of Q would maximize the firm's profits. [Note: your answer will not be a number. It will be a function.]

The First Order condition of profit maximization with respect to output yields

$$\frac{d\Pi}{dQ} = \alpha I - \frac{\beta}{N} Q - \frac{\beta}{N} Q - MC = 0 \quad \Rightarrow$$

$$\alpha I - MC = 2 \frac{\beta}{N} Q \quad \Rightarrow$$

$$Q = \frac{N}{2} \frac{\alpha I - MC}{\beta}$$

c. What is the corresponding market price? [Note: your answer will be a function.]

Plugging the optimal output level into the demand function we get

$$P = \alpha I - \frac{\beta}{N}\frac{N}{2}\frac{\alpha I - MC}{\beta}$$

$$= \alpha I - \frac{1}{2}(\alpha I - MC)$$

$$= \frac{\alpha I + MC}{2}$$

d. Would the firm raise its price if per capita income increases?

Yes, the optimal price is increasing in the per capital income.

e. Would the firm charge a higher price in bigger markets, i.e., in markets that have a larger number of kids of the appropriate age?

No, the optimal price is independent of the market size.

3. A firm is selling its output in two different markets. The demand function for market 1 is given by

$$P_1 = 100 - 20\ Q_1$$

The demand function for market 2 is given by

$$P_2 = 80 - 10\ Q_2$$

This firm has no fixed costs. The marginal cost of selling in market 1 equals c_1 while the marginal cost of selling in market 2 equals c_2. [These costs include both the production and delivery cost of the good.] The two costs can be different because the cost of shipping the good can differ across markets.

This firm is the sole producer of the good. Therefore, the price in each of the markets is determined by the amount that it decides to sell there. The firm chooses the quantity to sell in each market to maximize its total profits from both markets.

a. Write down the profit function of the firm in terms of its decision variables. [Note: your answer will be a function, not a number. It will depend on the marginal costs of the firm.]

The profits of the firm are equal to the sum of the profits that it makes in each market. Because the firm has no fixed costs and constant (for each market) marginal costs, the profits of the firm equal:

$$\Pi = (P_1 - c_1)\ Q_1 + (P_2 - c_2)\ Q_2$$

However, the price in each of the markets depends on how much the firm sells there. Substituting in for the price as a function of the quantity sold in the market we get:

$$\Pi = (100 - 20\ Q_1 - c_1)\ Q_1 + (80 - 10\ Q_1 - c_2)\ Q_2$$

b. What choice of quantities for each of the two markets will maximize the firm's profits?

Maximizing the profit function with respect to the output sold in the two markets yields the First Order Conditions

$$\frac{\partial \Pi}{\partial Q_1} = -\ 20\ Q_1 + 100 - 20\ Q_1 - c_1 = 0$$

and

$$\frac{\partial \Pi}{\partial Q_2} = -10\ Q_2 + 80 - 10\ Q_2 - c_2 = 0$$

Solving the first equation for Q_1 yields

$$40\ Q_1 = 100 - c_1 \quad \Rightarrow$$

$$Q_1 = \frac{100 - c_1}{40}$$

Similarly, solving the second equation for Q_2 yields

$$20\ Q_2 = 80 - c_2 \quad \Rightarrow$$

$$Q_2 = \frac{80 - c_2}{20}$$

c. What will the corresponding prices in each of the markets be?

In market 1 the price that corresponds to the profit maximizing quantity level is

$$P_1 = 100 - 20\ Q_1$$

$$= 100 - 20\ \frac{100 - c_1}{40}$$

$$= \frac{100 + c_1}{2}$$

In market 2 the price that corresponds to the profit maximizing quantity level is

$$P_2 = 80 - 10\ Q_2$$

$$= 80 - 10\ \frac{80 - c_2}{20}$$

$$= \frac{80 + c_2}{2}$$

4. A firm produces memory sticks with 500MB capacity, at a cost of $20 each. The current price of the sticks is $60 and the firm sells 2,000 units per day. The firm is concerned that it is mis-pricing its product, and that it could make more money by charging a different price. Your team is hired as consultants to evaluate the demand for memory sticks and suggest the appropriate price.

a. Using data on prices of similar products in different countries, and different points in time, you team estimates that the daily demand for memory sticks is $P = 80 - 0.01\ Q$. What price would you suggest that the firm charges to maximize its profits?

We have seen that if there is a single firm facing a (deterministic) downward sloping demand curve, then the profit maximizing price is the same regardless of whether the firm is choosing output to maximize profits, or it is choosing price to maximize profits. Therefore, we will formulate the problem in terms of optimal output choice. The optimal output would satisfy the condition that marginal revenue is equal to marginal cost. Since the demand curve is linear, marginal revenue is a straight line with the same intercept as that of the demand curve and twice the slope. That is, $MR = 80 - 0.02\, Q$. Therefore, the optimal output will satisfy the equation

$$80 - 0.02\, Q = 20 \Rightarrow$$

$$Q = \frac{60}{0.02} \Rightarrow$$

$$Q = 3000$$

The corresponding price is given by

$$P = 80 - 0.01\ 3000$$
$$= 50$$

Therefore, the firm will maximize profits by reducing its price to $50.

b. What is the increase in the firm's profits that arises from your consulting advise? If the consulting fee is 1 million dollars, how many days will it take for the firm to recoup the fee from the gain in profits? (assume no discounting).

When the firm charged a price of 60 and sold 2000 units, its daily profit was

$$\Pi_0 = P\ Q - C(Q)$$
$$= 60\ 2{,}000 - 20\ 2{,}000$$
$$= 120{,}000 - 40{,}000$$
$$= 80{,}000$$

With the recommended price of 50, the firm's profits are

$$\Pi_1 = 50\ 3{,}000 - 20\ 3{,}000$$
$$= 150{,}000 - 60{,}000$$
$$= 90{,}000$$

Your advise results in an increase in the daily profit of $10,000. Since your consulting fee was 1 million dollars, the firm will recoup this fee after 100 days.

The firm is now charging the price you have suggested in part (a). Some time later, the engineering staff of the firm comes to the firm's management and says that they can produce memory sticks with 1000MB capacity at a cost of $30 each. They argue that if the firm passes the increase in costs to the consumers, the $10 price increase for twice the capacity will surely be a better deal for consumers than the current sticks. Thus, they argue that more people are likely to buy the new sticks, leading to higher profits for the firm.

The firm has no idea about what the how demand will be affected by the increased capacity. It also has no idea whether it should pass the entire cost increase to the consumers, or increase its price by more or less. It again hires your consultancy to give advice about whether they should replace the old memory sticks with the old ones, and how to price the new sticks if they should introduce them.

c. Using information from the sale of other products and the usage patterns of storage devices, your team estimates that demand for the new sticks will be given by $P = 80 - \frac{1}{120} Q$. Would you recommend that the firm raises its price if it were to introduce the new memory sticks? If so, then by how much?

If the firm introduced the new memory sticks, both the demand for its product, and the marginal cost of production would change. Therefore, the optimal price and quantity sold would change. The optimal output (quantity sold) would still be given by the condition $MR = MC$.

This now yields the equation

$$80 - \frac{1}{60} Q = 30$$

Solving for output, we obtain

$$4{,}800 - Q = 1{,}800 \quad \Rightarrow$$

$$Q = 3{,}000$$

Therefore, the optimal price is

$$P = 80 - \frac{1}{120} 3{,}000$$

$$= 80 - 25$$

$$= 55$$

The optimal output of the firm is unchanged, but the firm would charge a higher price.

d. Would you recommend against or in favor of the firm introducing the new memory sticks? Show your work underlying your conclusion.

To determine whether the firm should introduce the new memory sticks or not, one would have to calculate the profits of the firm if it were to introduce the new memory sticks and charge the optimal price, and compare them with Π_1. The profits the firm would earn if it were to introduce the memory sticks are

$$\Pi_2 = 55\ 3{,}000 - 30\ 3{,}000$$

$$= 25\ 3{,}000$$

$$= 75{,}000$$

This profit level is less than \$90,000. Thus, you should recommend against introducing the new memory sticks.

e. How would your recommendation about pricing and product introduction differ from that of the engineering staff? What would have happened if the firm followed the engineering staff's advise to the letter?

You would recommend against rather than in favor of introducing the product. The argument of the engineers may sound plausible but it turns out not be correct. In this particular case, an optimal increase in the price would not sacrifice any sales, but would fail to fully compensate for the increase in costs. Moreover, if the firm were to introduce the new memory sticks, you would have advised them to pass only a portion of the cost increase to consumers. Increasing price by \$10 dollars would actually lead to even lower profits than increasing it by \$5.