Short Questions

1. Consider a firm with production function given by

$$Q = 4G + 2F$$

where G is the amount of natural gas used and F is the amount of fuel oil used. The firm wants to produce 20 units of output, and the current input prices are $W_G = 3$ and $W_F = 5$.

A. What choice of G and F minimizes the firm's costs? Show how you obtain your answer either by using a diagram of an isoquant and isocosts or through mathematical expressions.

The easiest way is to solve this problem is to recall that when a production process is characterized by linear production function, then the firm will use only one input, the one that gives the biggest marginal product per dollar spent (or biggest "bang-for-the-buck").

The marginal product of natural gas per dollar spent is

$$\frac{MP_G}{w_G} = \frac{4}{3}$$

The marginal product fuel oil per dollar spent is

$$\frac{MP_F}{w_F} = \frac{2}{5}$$

Clearly, natural gas is more productive per dollar spent, and the firm will produce the output only with natural gas. The quantity of natural gas the firm will need to produce 20 units is obtained by solving the equation

$$20 = 4G$$

for G, which yields

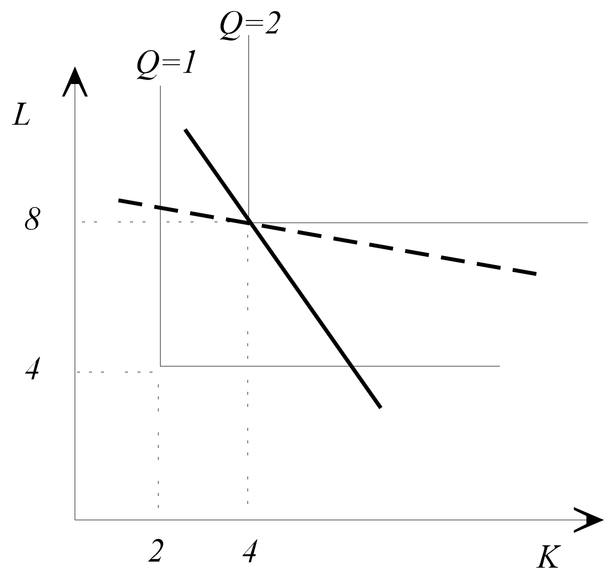
$$G = 5$$

B. Does the "no arbitrage in input use" condition hold at the optimum?

Clearly it does not hold since

$$\frac{MP_G}{w_G} > \frac{MP_F}{w_F}$$

2. Consider a firm whose production function is characterized by the following isoquants.



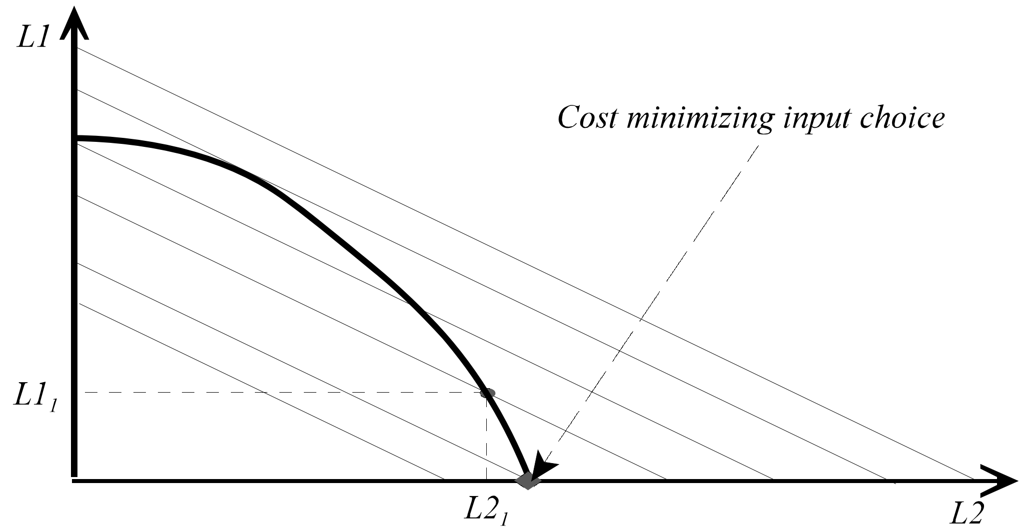
A. What is the cost minimizing input choice that would allow the firm to produce two units of output if the price of labor is 2 and the price of capital is 4?

Regardless of the slope of the isocost lines the cost-minimizing input choice that allows the firm to produce two units of output is 4 units of *K* and 8 units of *L* (see the figure above in which the bold solid and bold dashed lines are two different isocosts corresponding to different relative prices of *K* and *L*).

B. How would an increase in the price of labor to 4 change the above answer?

Given the discussion above and the isocosts drawn in the figure, the cost-minimizing choice of inputs would not be affected if the price of labor increased to 4.

3. One isoquant (the thick curve) and a few isocosts (thin straight lines) are plotted in the figure below. The inputs consist of two different types of labor, L1 and L2.



a. In the above figure, clearly indicate and label the cost minimizing choice of inputs that are required to produce the output that corresponds to the isoquant given.

The optimum corresponds to that small diamond in the above picture. The firm will only use labor input L2 to produce the output, and do not use any of labor input L1.

b. Which of the following expressions are (or is) true at the cost minimizing choice of inputs? Circle the ones that are known to be correct on the basis of the above figure.

$$\boxed{\frac{MP_{L1}}{w_{L1}} < \frac{MP_{L2}}{w_{L2}}} \qquad \frac{MP_{L1}}{w_{L1}} = \frac{MP_{L2}}{w_{L2}} \qquad \frac{MP_{L1}}{w_{L1}} > \frac{MP_{L2}}{w_{L2}}$$

Because L2 is the only input the firm is using, the optimal input choice is a corner solution, and therefore input L2 must yield a higher marginal product per dollar spent on it.

$$\boxed{MP_{L1} < MP_{L2}} \qquad MP_{L1} = MP_{L2} \qquad MP_{L1} > MP_{L2}$$

The slope of the isoquant gives the MRTS between the two inputs, i.e., it gives the ratio of the marginal products. The isoquant is steeper than 45-degrees at cost minimizing point. Thus, it takes more than 1 unit of L1 to make up for the loss of one units of L2, and therefore, the marginal product of L2 must exceed the marginal product of L1.

$w_{L1} < w_{L2}$ $w_{L1} = w_{L2}$ $w_{L1} > w_{L2}$

The relative prices of the inputs are given by the slope of the isocosts. The isocosts have a slope of less then -1 (less than 45 degrees) which means L1 is more expensive than L2. You can also see this from the intercepts: the intercept for L2 is further away from the origin than the intercept for L1, which means that for the same amount of expenditure the firm can afford of L2 than of L1.

c. How would this optimal choice of inputs change if the price of L2 decreases by a little bit? Would the utilization of L2 increase, decrease, or stay the same? How would the utilization of L1 change?

L2 is already a better deal (as an input) than L1. If it becomes cheaper, the firm will still only use L2. Nothing will change.

d. Answer the questions in (c) if instead the price of L2 increased by a little bit?

A small increase in the price of L2 will also not lead to any changes, because the isocost will remain flatter than the isoquant at the "corner solution". Only a relative large change in prices will lead the firm to make a change in its input choice.

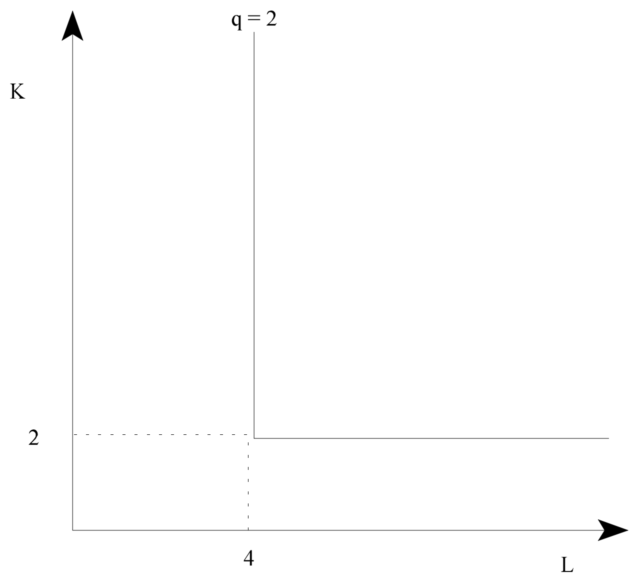
e. How much would the price of L2 have to change for the optimal input use to be given by $L1_1$ and $L2_1$? Draw another isocost if needed to illustrate your point, or explain verbally (it should only take a sentence or two).

No set of prices will make the firm choose input combinations $L1_1$ and $L2_1$. No matter what the slope of an isocost that goes through that point, there will be other points of the isoquant that would be to the left of that isocost, i.e., there will be input combinations that cost less. In fact, for isoquants like this one that exhibit Increasing MRTS, the cost minimizing input choices will be corner solutions.

Problems

1. A firm has a fixed proportions production function where 2 units of labor and one unit of capital are used to produce one unit of output.

a. Draw the isoquant that corresponds to 2 units of output, with capital on the vertical axis and labor on the horizontal axis. Label all important points.



b. Suppose the cost of labor is 3 and the cost of capital 2. What is the cost of producing 2 units of output? What is the cost of producing q units of output?

$$Cost = w\ L + r\ K$$

Since for every unit of output we need 2 units of labor and one of capital, the cost of producing 2 units is $3\ 2\ 2 + 2\ 2\ 1 = 12 + 4 = 16$. The cost of producing q units is:

$$C(q) = 3\ 2\ q + 2\ 1\ q \quad \Rightarrow$$

$$C(q) = 6\ q + 2\ q \quad \Rightarrow$$

$$C(q) = 8\ q$$

c. What is the marginal cost of the firm ?

$$MC = \frac{dC(q)}{dq} = 8$$

d. What is the average cost of the firm ?

$$AC = \frac{C(q)}{q} = 8$$

e. If this firm is a monopolist and faces a demand curve P = 50 - q what is the profit maximizing choice of output?

Maximizing profit requires increasing output until marginal revenue is equal marginal cost. The marginal revenue of a linear demand curve is a line with the same intercept and twice the slope. Hence,

$$MR = MC \quad \Rightarrow$$

$$50 - 2q = 8 \quad \Rightarrow$$

$$2q = 42 \quad \Rightarrow$$

$$q = 21$$

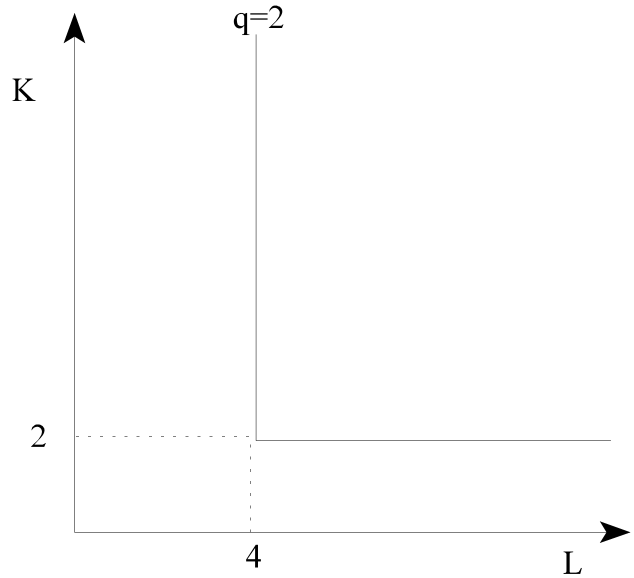
f. What is the corresponding market price?

Plugging in the profit maximizing choice of output into the demand function will yield the profit maximizing choice of output.

$$P = 50 - 21$$
$$= 29$$

2. A firm has a fixed proportions production function where 2 units of labor and one unit of capital are used to produce one unit of output.

a. Draw the isoquant that corresponds to 2 units of output, with capital on the vertical axis and labor on the horizontal axis. Label all important points.



b. Suppose the cost of labor is *w* and the cost of capital 2. What is the cost of producing 2 units of output? What is the cost of producing q units of output ? [Parts (b.) through (f.) will have answers that are functions of *w*.]

The cost of producing 2 units is the cost of hiring 4 units of labor and 2 units of capital. This is equal to $4\,w + 2\ 2 = 4\,w + 4$. In general, the cost of producing q units of output is the cost of hiring $2\,q$ units of labor and q units of capital. Therefore,

$$C(q) = 2\ q\ w + 2\ q$$
$$= (w + 1)\ 2\ q$$

c. What is the marginal cost of the firm?

$$MC = \frac{dC(q)}{dq}$$
$$= 2\ (w + 1)$$

d. What is the average cost of the firm?

$$AC = \frac{C(q)}{q}$$
$$= 2\ (w + 1)$$

e. If this firm is a monopolist and faces a demand curve P = 50 - *q* what is the profit maximizing choice of output?

The monopolist will increase out until marginal revenue is equal to marginal cost. The marginal revenue associated with a linear demand curve is a line with the same intercept as the demand and twice the slope. Therefore, we have:

$$MR = MC \quad \Rightarrow$$

$$50 - 2\ q = 2\ (w + 1) \quad \Rightarrow$$

$$50 - 2\ (w + 1) = 2\ q \quad \Rightarrow$$

$$q = 25 - w - 1 \quad \Rightarrow$$

$$q = 24 - w$$

f. What is the corresponding market price ?

If we substitute the optimal choice of output into the demand curve, we will find the corresponding optimal price for the monopoly.

$$P = 50 - (24 - w)$$
$$= 26 + w$$

g. How much does a dollar increase in the cost of labor affect the market price?

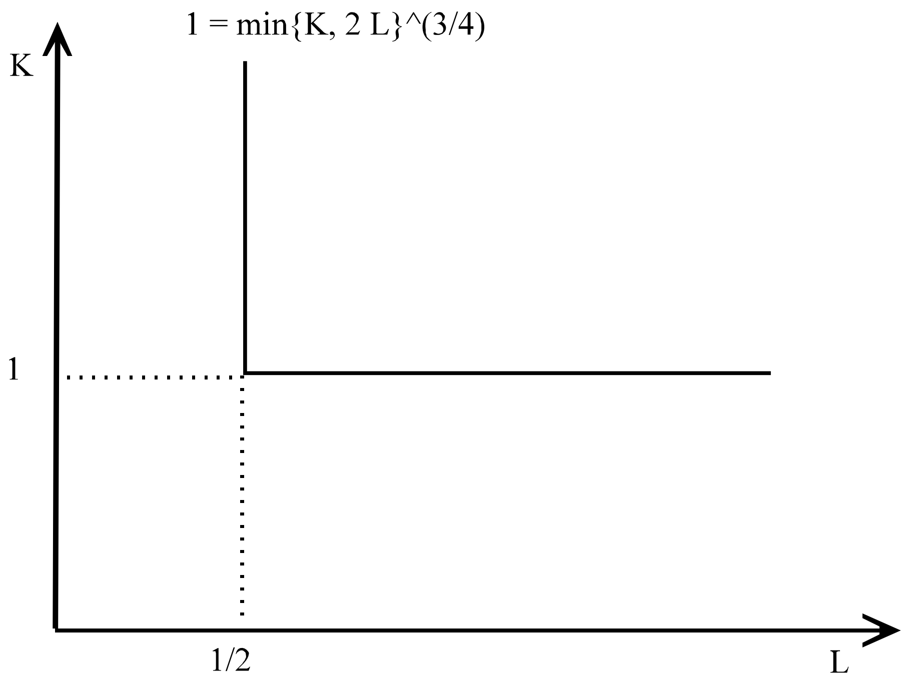
We can readily see from the answer to part (f) that in this example an increase of the wage by a dollar will increase the optimal monopoly price by a dollar.

3. A firm has a Leontieff production function given by

$$q = \min\{K, 2L\}^{\frac{3}{4}}$$

a. Draw the isoquant that corresponds to 1 unit of output, with capital on the vertical axis and labor on the horizontal axis. Label all important points.

One unit of output would require 1 unit of *K* and ½ units of *L*. Additional *L* or *K* would not increase output at all. Therefore, the isoquant that corresponds to *q*=1 will have the shape given below:



b. Suppose the price of labor is 3 and the price of capital 2. What is the cost of producing *q* units of output?

Because this is a fixed-proportions production function, the required capital and labor to produce *q* units of output are, respectively, given by

$$K(q) = q^{\frac{4}{3}} \qquad \text{and} \qquad L(q) = \frac{1}{2} L^{\frac{4}{3}}$$

The cost of producing q units, then, equals

$$C(q) = w_K \, K(q) + w_L \, L(q)$$

$$= 2 \, q^{\frac{4}{3}} + 3 \, \frac{1}{2} \, q^{\frac{4}{3}}$$

$$= \frac{7}{2} \, q^{\frac{4}{3}}$$

c. What is the marginal cost of the firm?

Marginal cost is the slope of the cost function with respect to output. Therefore,

$$MC(q) = \frac{4}{3} \, \frac{7}{2} \, q^{\frac{1}{3}}$$

$$= \frac{14}{3} \, q^{\frac{1}{3}}$$

d. What is the average cost of the firm?

Average cost is the ratio of total cost to output. Therefore,

$$AC(q) = \frac{7}{2} \, q^{\frac{1}{3}}$$

e. If this firm faces a horizontal demand curve at a price P, what is the profit maximizing choice of output?

The profit maximizing choice of output is that for which $MR(q) = MC(q)$, which for horizontal demand curves is equivalent to $P = MC(q)$. Therefore, for this firm, the optimal choice of output is given by

$$P = \frac{14}{3} \, q^{\frac{1}{3}} \quad \Rightarrow$$

$$q = \left(\frac{3}{14}\right)^3 P^3$$