Short Questions

1. A consulting firm has three inputs, staff (S), managers (M), and office equipment (E). The firm's production function of consulting services using these three inputs is given by

$$q = \sqrt{S} + 3\sqrt{M} + \sqrt{E}$$

The price of office equipment is 1, the wage of the staff is 2, while the wage of managers is 10.

Write the system of equations that you would use to calculate the values of S, M, and E that would allow the firm to produce output q at the lowest possible cost. [Hint: The firm's production function is differentiable, exhibits diminishing *MRTS*, and the prices are such that all three inputs will be used.]

Note: You do not have to calculate the optimal values of S, M, and E, but you need to write the equations in terms of S, M, E, and q.

The "hint" above says that this production function does not belong to any of the three exceptions to using the Lagrangian approach to cost minimization. This approach yields the no-arbitrage in input choice condition that the ratio of marginal products to input prices must the same for all three inputs. Therefore, cost minimization requires

$$\frac{MP_S}{w_S} = \frac{MP_M}{w_M} = \frac{MP_E}{w_E}$$

which yields the two equations

$$\frac{\frac{1}{2}\frac{1}{\sqrt{S}}}{2} = \frac{\frac{1}{2}\frac{3}{\sqrt{M}}}{10} \quad \text{and} \quad \frac{\frac{1}{2}\frac{1}{\sqrt{S}}}{2} = \frac{\frac{1}{2}\frac{1}{\sqrt{E}}}{1}$$

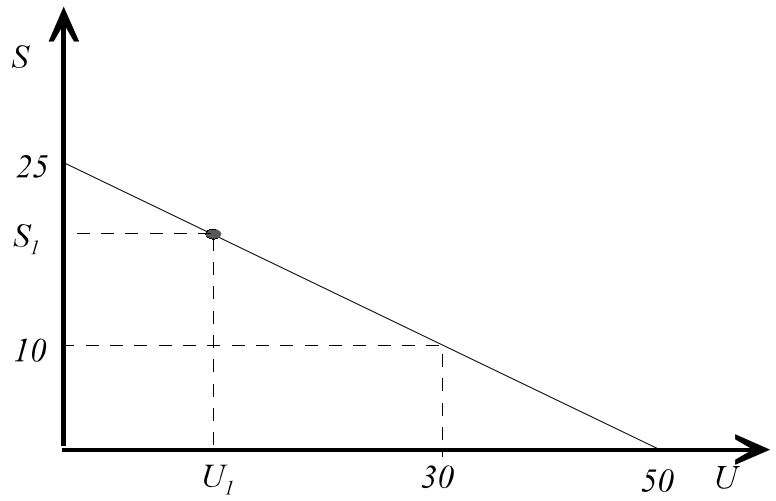
which simplify to

$$\frac{1}{4\sqrt{S}} = \frac{3}{20\sqrt{M}} \quad \text{and} \quad \frac{1}{4\sqrt{S}} = \frac{1}{2\sqrt{E}}$$

The third equation is the production function itself, which is the constraint.

$$q = \sqrt{S} + 3\sqrt{M} + \sqrt{E}$$

2. The following figure plots an isocost for skilled labor (S) and unskilled labor (U). Skilled labor has a wage of 40 while unskilled labor has a wage of 20.



a. What is the cost of the firm of hiring the amount of skilled and unskilled labor that corresponds to the dot in the above isocost? In other words, what is the cost of hiring U_1 hours of unskilled labor and S_1 hours of skilled labor? Your answer should be a number.

Any point on the isocost has the same cost. Therefore, U_1 hours of unskilled labor and S_1 hours of skilled labor cost the same as 10 hours of skilled labor and 30 hours of unskilled labor. The cost of this combination of skilled and unskilled labor is equal to

$$C = 40 * 10 + 20 * 30 = 400 + 600 = 1000$$

b. Write in the above figure the numerical value of the S-axis and U-axis intercepts.

The S-axis intercept is the amount of skilled labor the firm could hire it spent the entire budget the corresponds to this isocost on skilled labor. This is equal to 1,000/40=25.

The U-axis intercept is the amount of unskilled labor the firm would hire if it spent the entire budget on unskilled labor. This is equal to 1,000/20 = 50.

c. Draw in the above figure the isocost that corresponds to an expenditure of 400. Is it steeper, flatter, or parallel to the isocost that is already drawn above? Label the two intercepts clearly.

The easiest way to draw this isocost is to plot the two intercepts and then connect them, while keeping in mind that this isocost should be parallel to the one already drawn in the figure. An expenditure of 400 will allow the firm to hire 10 units of skilled labor and 20 units of unskilled labor. Hence, the isocost that corresponds to expenditure of 400 is given by the thick solid line in the figure above.

Problems

None for this lecture.