Chapter 5: Answers to Questions and Problems

1.

a. When $K = 16$ and $L = 16$, $Q = (16)^{0.75}(16)^{0.25} = 16$. Thus, $AP_L = Q/L = 16/16 = 1$. When $K = 16$ and $L = 81$, $Q = (16)^{0.75}(81)^{0.25} = (8)(3) = 24$. Thus, $AP_L = 24/81 = 8/27$.

b. The marginal product of labor is $MP_L = 2(L)^{-3/4}$. When $L = 16$, $MP_L = 2(16)^{-3/4} = 1/4$. When $L = 81$, $MP_L = 2(81)^{-3/4} = 2/27$. Thus, as the number of units of labor hired increases, the marginal product of labor decreases $MP_L(16) = 1/4 > 2/27 = MP_L(81)$, holding the level of capital fixed.

c. We must equate the value marginal product of labor equal to the wage and solve for L. Here, $VMP_L = (P)(MP_L) = (\$100)\left(2(L)^{-3/4}\right) = 200(L)^{-3/4}$. Setting this equal to the wage of \$25 gives $200(L)^{-3/4} = 25$. Solving for L, the optimal quantity of labor is $L = 16$.

2. See Table 5-1.

(1) Capital K	(2) Labor L	(3) Output Q	(4) Marginal Product of Capital MP_K	(5) Average Product of Capital AP_K	(6) Average Product of Labor AP_L	(7) Value Marginal Product of Capital VMP_K
0	20	0	--	--	--	--
1	20	50	50	50	2.50	100
2	20	150	100	75	7.50	200
3	20	300	150	100	15	300
4	20	400	100	100	20	200
5	20	450	50	90	22.50	100
6	20	475	25	79.17	23.75	50
7	20	475	0	67.86	23.75	0
8	20	450	-25	56.25	22.50	-50
9	20	400	-50	44.44	20	-100
10	20	300	-100	30	15	-200
11	20	150	-150	13.64	7.50	-300

Table 5-1

a. Labor is the fixed input while capital is the variable input.
b. Fixed costs are 20(\$15) = \$300.
c. To produce 475 units in the least-cost manner requires 6 units of capital, which cost \$75 each. Thus, variable costs are (\$75)(6) = \$450.
d. Using the $VMP_K = r$ rule, $K = 5$ maximizes profits.
e. The maximum profits are $\$2(450) - \$15(20) - \$75(5) = \$225$.

f. There are increasing marginal returns when K is between 0 and 3.
g. There are decreasing marginal returns when K is between 3 and 11.
h. There are negative marginal returns when K is greater than 7.

3. The law of diminishing marginal returns is the decline in marginal productivity experienced when input usage increases, holding all other inputs constant. In contrast, the law of diminishing marginal rate of technical substitution is a property of a production function stating that as less of one input is used, increasing amounts of another input must be employed to produce the same level of output.

4.

a. FC = 50
b. $VC(10) = 25(10) + 30(10)^2 + 5(10)^3 = \$8,250$.
c. $C(10) = 50 + 25(10) + 30(10)^2 + 5(10)^3 = \$8{,}300$.
d. $AFC(10) = \frac{\$50}{10} = \$5$.
e. $AVC(10) = \frac{VC(10)}{10} = \frac{\$8,250}{10} = \$825$.
f. $ATC(10) = AFC(10) + AVC(10) = \$830$.
g. $MC(10) = 25 + 60(10) + 15(10)^2 = \$2{,}125$.

5. Since $MRTS_{KL} \neq \frac{w}{r}$, the firm is not using the cost minimizing combination of labor and capital. To minimize costs, the firm should use more labor and less capital since the marginal product per dollar spent is greater for labor: $\frac{MP_L}{w} = \frac{50}{6} > \frac{MP_K}{r} = \frac{75}{12}$.

6. See Table 5-2.

(1) Quantity *Q*	(2) Fixed Cost *FC*	(3) Variable Cost *VC*	(4) Total Cost *TC*	(5) Average Fixed Cost *AFC*	(6) Average Variable Cost *AVC*	(7) Average Total Cost *ATC*	(8) Marginal Cost *MC*
0	10,000	0	10,000	--	--	--	--
100	10,000	10,000	20,000	100	100	200	100
200	10,000	15,000	25,000	50	75	125	50
300	10,000	30,000	40,000	33.33	100	133.33	150
400	10,000	50,000	60,000	25	125	150	200
500	10,000	90,000	100,000	20	180	200	400
600	10,000	140,000	150,000	16.67	233.33	250	500

Table 5-2

7.

a. For a quadratic multi-product cost function, economies of scope exist if $f - aQ_1Q_2 > 0$. In this case, $f = 75$ and $a = -0.25$, so economies of scope exist since *f* is fixed cost, which is always nonnegative.

b. Cost complementarities exist since $a = -0.25 < 0$.

c. Since $a = -0.25 < 0$, the marginal cost of producing product 1 will increase if the division that produces product 2 is sold.

8. Fixed costs are associated with fixed inputs, and do not change when output changes. Variable costs are costs associated with variable inputs, and do change when output changes. Sunk costs are costs that are forever lost once they have been paid.

9. An investment tax credit would reduce the relative price of capital to labor. Other things equal, this would increase $\frac{w}{r}$, thereby making the isocost line more steep. This means that the cost-minimizing input mix will now involve more capital and less labor, as firms substitute toward capital. Labor unions are likely to oppose the investment tax credit since the higher capital-to-labor ratio will translate into lost jobs. You might counter this argument by noting that, while some jobs will be lost due to substituting capital for labor, many workers will retain their jobs. Absent the plan, automakers have an incentive to substitute cheaper foreign labor for U.S. labor. The result of this substitution would be a movement of plants abroad, resulting in the complete loss of U.S. jobs.

10. Since $MRTS_{KL} \neq \frac{w}{r}$, the firm was not using the cost minimizing combination of labor and capital. To achieve the cost minimizing combination of inputs, the previous manager should have used fewer units of capital and more units of labor, since $\frac{MP_L}{w} = \frac{100}{8} > \frac{MP_K}{r} = \frac{100}{16}$.

11. The profit-maximizing level of labor and output is achieved where $VMP_L = w$. Here, $VMP_L = 2(\$100)(4)^{1/2}(L)^{-1/2} = \$400(L)^{-1/2}$ and $w = \$100$ per day. Solving yields *L* = 16. The profit-maximizing level of output is $Q = 2(4)^{1/2}(16)^{1/2} = 16$ units. The firm's fixed costs are $10,000, its variable costs are $100(16) = $1,600, and its total revenues are $200(16) = $3,200. Profits are $3,200 – $11,600 = – $8,400. The firm is suffering a loss, but the loss is lower than the $10,000 that would be lost if the firm shut down its operation.

12. The higher wage rate in Europe induces Airbus to employ a more capital intensive input mix than Boeing. Since Airbus optimally uses fewer workers than Boeing, and profit-maximization entails input usage in the range of diminishing marginal product, it follows that the lower quantity of labor used by Airbus translates into a higher marginal product of labor at Airbus than at Boeing.

13. Table 5-3 provides some useful information for making your decision. According to the $VMP_L = w$ rule, you should hire five units of labor and produce 90 units of output to maximize profits. Your fixed costs are ($10)(5) = $50, your variable costs are ($50)(5) =$250, and your revenues are ($5)(90) = $450. Thus, your maximum profits are $450 - $300 = $150.

(1) **Labor** *L*	(2) **Capital** *K*	(3) **Output** *Q*	(4) **Marginal Product of Labor** MP_L	(5) **Average Product of Labor** AP_L	(6) **Average Product of Capital** AP_K	(7) **Value Marginal Product of Labor** VMP_L
0	5	0	--	--	--	--
1	5	10	10	10	2	50
2	5	30	20	15	6	100
3	5	60	30	20	12	150
4	5	80	20	20	16	100
5	5	90	10	18	18	50
6	5	95	5	15.8	19	25
7	5	95	0	13.6	19	0
8	5	90	-5	11.3	18	-25
9	5	80	-10	8.9	16	-50
10	5	60	-20	6	12	-100
11	5	30	-30	2.7	6	-150

Table 5-3

14. The $1,200 per month that could be earned by renting out the excess rental space.

15. Had she not spent the $6,000 on advertising but instead collected the $65,000 refund, her total loss would have been limited to her sunk costs of $10,000. Her decision to spend $6,000 on advertising in an attempt to fetch an extra $5,000 was clearly foolish. However, the $6,000 is a sunk cost and therefore irrelevant in deciding whether to accept the $66,000 offer. She should accept the $66,000 offer because doing so makes her $1,000 better off than obtaining the $65,000 refund.

16. Facility "L" produces 6 million kilowatt hours of electricity at the lowest average total cost, so this is the optimal facility for South-Florida. Facility "M" produces 2 million kilowatt hours of electricity at the lowest average total cost, so this is the optimal facility for the Panhandle. There are economies of scale up to about 3 million kilowatts per hour, and diseconomies of scale thereafter. Therefore, facility "M" will be operating in the range of economies of scale while facility "L" will be operating in the range of diseconomies of scale.

17. To maximize profits the firm should continue adding workers so long as the value marginal product of labor exceeds the wage. The value marginal product of labor is defined as the marginal product of labor times the price of output. Here, output sells for $50 per panel, so the value marginal product of the third worker is $50(290) = $14,500. Table 5-4 summarizes the VMP_L for each choice of labor. Since the wage is $7,000, the profit maximizing number of workers is 4.

Machines	**Workers**	**Output**	**MP_L**	**VMP_L**	**Wage**
5	0	0	–	–	–
5	1	600	600	$30,000	$7,000
5	2	1,000	400	$20,000	$7,000
5	3	1,290	290	$14,500	$7,000
5	4	1,480	190	$9,500	$7,000
5	5	1,600	120	$6,000	$7,000
5	6	1,680	80	$4,000	$7,000

Table 5-4

18. The rental rate of capital is ¥475,000, computed as $r = MP_K \times P = .5 \times 950{,}000 = 475{,}00$. Therefore, the marginal product of labor is 0.0014 cars per hour, which is found by solving $\frac{MP_L}{1{,}330} = \frac{0.5}{475{,}000}$. Costs are minimized when the marginal rate of technical substitution is 0.0028.

19. Given the tightly woven marine engine and shipbuilding divisions, economies of scope and cost complementarities are likely to exist. Eliminating the unprofitable marine engine division may actually raise the shipbuilding division's costs and cause that division to become unprofitable. For this argument to withstand criticism, you must show the CEO that the quadratic multi-product cost function exhibits cost complementarities and economies of scope, which occurs when $a < 0$ and $f - aQ_1Q_2 > 0$, respectively, and compare profitability under the different scenarios.