Chapter 3: Answers to Questions and Problems

1.

a. When P = $12, R = ($12)(1) = $12. When P = $10, R = ($10)(2) = $20. Thus, the price decrease results in an $8 increase in total revenue, so demand is elastic over this range of prices.

b. When P = $4, R = ($4)(5) = $20. When P = $2, R = ($2)(6) = $12. Thus, the price decrease results in an $8 decrease total revenue, so demand is inelastic over this range of prices.

c. Recall that total revenue is maximized at the point where demand is unitary elastic. We also know that marginal revenue is zero at this point. For a linear demand curve, marginal revenue lies halfway between the demand curve and the vertical axis. In this case, marginal revenue is a line starting at a price of $14 and intersecting the quantity axis at a value of Q = 3.5. Thus, marginal revenue is 0 at 3.5 units, which corresponds to a price of $7 as shown below.

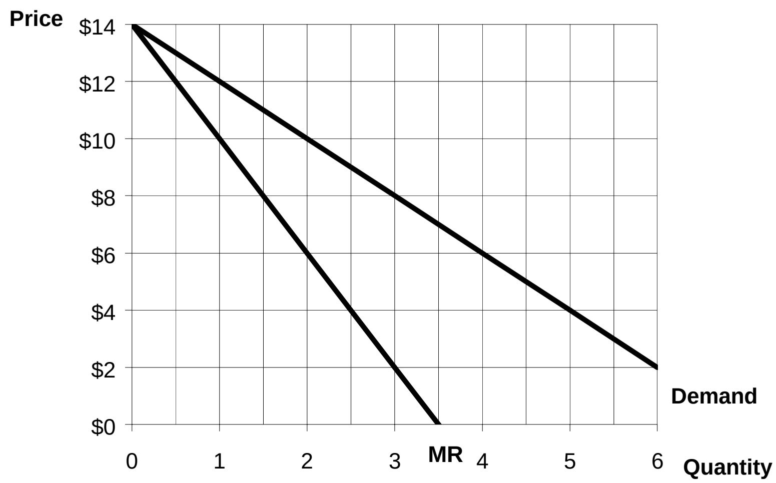

Figure 3-1

2.

a. At the given prices, quantity demanded is 700 units: $Q_x^d = 1000 - 2(154) + .02(400) = 700$. Substituting the relevant information into the elasticity formula gives: $E_{Q_x,P_x} = -2\frac{P_x}{Q_x} = -2\frac{154}{700} = -0.44$. Since this is less than one in absolute value, demand is inelastic at this price. If the firm charged a lower price, total revenue would decrease.

b. At the given prices, quantity demanded is 300 units: $Q_x^d = 1000 - 2(354) + .02(400) = 300$. Substituting the relevant information into the elasticity formula gives: $E_{Q_x,P_x} = -2\left(\frac{P_x}{Q_x}\right) = -2\left(\frac{354}{300}\right) = -2.36$. Since this is greater than one in absolute value, demand is elastic at this price. If the firm increased its price, total revenue would decrease.

c. At the given prices, quantity demanded is 700 units: $Q_x^d = 1000 - 2(154) + .02(400) = 700$. Substituting the relevant information into the elasticity formula gives: $E_{Q_x,P_Z} = .02\left(\frac{P_Z}{Q_x}\right) = .02\left(\frac{400}{700}\right) = 0.011$. Since this number is positive, goods X and Z are substitutes.

3.

a. The own price elasticity of demand is simply the coefficient of *ln* P_x, which is –0.5. Since this number is less than one in absolute value, demand is inelastic.

b. The cross-price elasticity of demand is simply the coefficient of *ln* P_y, which is –2.5. Since this number is negative, goods X and Y are complements.

c. The income elasticity of demand is simply the coefficient of *ln M*, which is 1. Since this number is positive, good X is a normal good.

d. The advertising elasticity of demand is simply the coefficient of *ln A*, which is 2.

4.

a. Use the own price elasticity of demand formula to write $\frac{\%\Delta Q_x^d}{5} = -2$. Solving, we see that the quantity demanded of good X will decrease by 10 percent if the price of good X increases by 5 percent.

b. Use the cross-price elasticity of demand formula to write $\frac{\%\Delta Q_x^d}{10} = -6$. Solving, we see that the demand for X will decrease by 60 percent if the price of good Y increases by 10 percent.

c. Use the formula for the advertising elasticity of demand to write $\frac{\%\Delta Q_x^d}{-2} = 4$. Solving, we see that the demand for good X will decrease by 8 percent if advertising decreases by 2 percent.

d. Use the income elasticity of demand formula to write $\frac{\%\Delta Q_x^d}{-3} = 3$. Solving, we see that the demand of good X will decrease by 9 percent if income decreases by 3 percent.

5. Using the cross price elasticity formula, $\frac{50}{\%\Delta P_y} = -5$. Solving, we see that the price of good Y would have to decrease by 10 percent in order to increase the consumption of good X by 50 percent.

6. Using the change in revenue formula for two products, $\Delta R = [\$30{,}000(1-2.5) + \$70{,}000(1.1)](.01) = \$320$. Thus, a 1 percent increase in the price of good X would cause revenues from both goods to increase by $320.

7. Table 3-1 contains the answers to the regression output.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.62
R Square	0.39
Adjusted R Square	0.37
Standard Error	190.90
Observations	100.00

ANOVA

	degrees of freedom	SS	MS	F	Significance F
Regression	2.00	2,223,017.77	1,111,508.88	30.50	0.00
Residual	97.00	3,535,019.49	36,443.50		
Total	99.00	5,758,037.26			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	187.15	534.71	0.35	0.73	-880.56	1,254.86
Price of X	-4.32	0.69	6.26	0.00	-5.69	-2.96
Income	0.09	0.02	4.47	0.00	0.05	0.14

Table 3-1

a. $Q_x^d = 187.15 - 4.32P_x + .09M$.

b. Only the coefficients for the Price of X and Income are statistically significant at the 5 percent level or better.

c. The *R*-square is fairly low, indicating that the model explains only 39 percent of the total variation in demand for X. The adjusted *R*-square is only marginally lower (37 percent), suggesting that the *R*-square is not the result of an excessive number of estimated coefficients relative to the sample size. The *F*-statistic, however, suggests that the overall regression is statistically significant at better than the 5 percent level.

8. The approximate 95 percent confidence interval for a is $\hat{a} \pm 2\sigma_{\hat{a}} = 10 \pm 2$. Thus, you can be 95 percent confident that a is within the range of 8 and 12. The approximate 95 percent confidence interval for b is $\hat{b} \pm 2\sigma_{\hat{b}} = -2.5 \pm 1$. Thus, you can be 95 percent confident that b is within the range of –3.5 and –1.5.

9. The result is not surprising. Given the available information, the own price elasticity of demand for Palm's brand of PDAs is $E_{Q,P} = \frac{137}{-17} = -8.06$. Since this number is greater than one in absolute value, demand is elastic. By the total revenue test, this means that a reduction in price will increase revenues.

10. The regression output is as follows:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.97
R Square	0.94
Adjusted R Square	0.94
Standard Error	0.00
Observations	49

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	0.00702	0.004	370.38	0.0000
Residual	46	0.00044	0.000		
Total	48	0.00745			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	1.29	0.41	3.12	0.00	0.46	2.12
LN Price	-0.07	0.00	-26.62	0.00	-0.08	-0.07
LN Income	-0.03	0.09	-0.33	0.74	-0.22	0.16

Table 3-2

Thus, the demand for your batteries is given by $\ln Q = 1.29 - 0.07 \ln P - 0.03 \ln M$. Since this is a log-linear demand equation, the best estimate of the income elasticity of demand for your product is -.03: Your batteries are an inferior good. However, note the estimated income elasticity is very close to zero (implying that a 3 percent reduction in global incomes would increase the demand for your product by less than one tenth of one percent). More importantly, the estimated income elasticity is not statistically different from zero (the 95 percent confidence interval ranges from a low of -.22 to a high of .16, with a t-statistic that is well below 2 in absolute value). On balance, this means that a 3 percent decline in global incomes is unlikely to impact the sales of your product. Note that the *R*-square is reasonably high, suggesting the model explains 94 percent of the total variation in the demand for this product. Likewise, the *F*-test indicates that the regression fit is highly significant.

11. Based on this information, the own price elasticity of demand for Big G cereal is $E_{Q,P} = \frac{-3}{2} = -1.5$. Thus, demand for Big G cereal is elastic (since this number is greater than one in absolute value). Since Lucky Charms is one particular brand of cereal for which even more substitutes exist, you would expect the demand for Lucky Charms to be even more elastic than the demand for Big G cereal. Thus, since the demand for Lucky Charms is elastic, one would predict that the increase in price of Lucky Charms resulted in a reduction in revenues on sales of Lucky Charms.

12. Use the income elasticity formula to write $\frac{\%\Delta Q^d}{-4} = 1.75$. Solving, we see that coffee purchases are expected to decrease by 7 percent.

13. To maximize revenue, GM should charge the price that makes demand unit elastic. Using the own price elasticity of demand formula, $E_{Q,P} = (-1.25)\left(\frac{P}{100,000 - 1.25P}\right) = -1$. Solving this equation for *P* implies that the revenue maximizing price is $P = \$40,000$.

14. Using the change in revenue formula for two products, $\Delta R = [\$600(1-2.5) + \$400(-0.2)] \times (-.01) = \$9.8 \text{ million}$, so revenues will increase by $9.8 million.

15. The estimated demand function for residential heating fuel is $Q^d_{RHF} = 136.96 - 91.69P_{RHF} + 43.88P_{NG} - 11.92P_E - 0.05M$, where P_{RHF} is the price of residential heating fuel, P_{NG} is the price of natural gas, P_E is the price of electricity, and M is income. However, notice that coefficients of income and the price of electricity are not statistically different from zero. Among other things, this means that the proposal to increase the price of electricity by $5 is unlikely to have a statistically significant impact on the demand for residential heating fuel. Since the coefficient of P_{RHF} is -91.69, a $2 increase in P_{RHF} would lead to a 183.38 unit reduction in the consumption of residential heating fuel (since (-91.69)($2) = - 183.38 units). Since the coefficient of P_{NG} is 43.88, a $1 reduction in P_{NG} would lead to a 43.88 unit reduction in the consumption of residential heating fuel (since (43.88)(-$1) = -43.88). Thus, the proposal to increase the price of residential heating fuel by $2 would lead to the greatest expected reduction in the consumption of residential heating fuel.

16. The regression output is as follows:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.97
R Square	0.94
Adjusted R Square	0.94
Standard Error	0.06
Observations	41

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2.24	2.24	599.26	0.00
Residual	39	0.15	0.00		
Total	40	2.38			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	4.29	0.12	37.17	0.00	4.06	4.53
ln (Price)	-1.38	0.06	-24.48	0.00	-1.50	-1.27

Table 3-3

Thus, the least squares regression line is $\ln Q = 4.29 - 1.38 \ln P$. The own price elasticity of demand for broilers is –1.38. From the *t*-statistic, this is statistically different from zero (the *t*-statistic is well over 2 in absolute value). The *R*-square is relatively high, suggesting that the model explains 94 percent of the total variation in the demand for chicken. Given that your current revenues are $750,000 and the elasticity of demand is –1.38, we may use the following formula to determine how much you must change price to increase revenues by $50,000:

$$\Delta R = \left[P_x \cdot Q_x \left(1 + E_{Q_x, P_x}\right)\right] \times \frac{\Delta P_x}{P_x}$$

$$\$50{,}000 = \left[\$750{,}000(1 - 1.38)\right] \frac{\Delta P_x}{P_x}$$

Solving yields $\frac{\Delta P_x}{P_x} = \frac{\$50{,}000}{-\$285{,}000} = -0.175$. That is, to increase revenues by $50,000, you must decrease your price by 17.5 percent.

17. The regression output (and corresponding demand equations) for each state are presented below:

ILLINOIS
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.29
R Square	0.09
Adjusted R Square	0.05
Standard Error	151.15
Observations	50

ANOVA

	degrees of freedom	*SS*	*MS*	*F*	*Significance F*
Regression	2	100540.93	50270.47	2.20	0.12
Residual	47	1073835.15	22847.56		
Total	49	1174376.08			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-42.65	496.56	-0.09	0.93	-1041.60	956.29
Price	2.62	13.99	0.19	0.85	-25.53	30.76
Income	14.32	6.83	2.10	0.04	0.58	28.05

Table 3-4

The estimated demand equation is $Q = -42.65 + 2.62P + 14.32M$. While it appears that demand slopes upward, note that coefficient on *price* is not statistically different from zero. An increase in *income* by $1,000 increases demand by 14.32 units. Since the *t*-statistic associated with *income* is greater than 2 in absolute value, *income* is a significant factor in determining quantity demanded. The *R*-square is extremely low, suggesting that the model explains only 9 percent of the total variation in the demand for KBC microbrews. Factors other than *price* and *income* play an important role in determining quantity demanded.

INDIANA
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.87
R Square	0.76
Adjusted R Square	0.75
Standard Error	3.94
Observations	50

ANOVA

	degrees of freedom	*SS*	*MS*	*F*	*Significance F*
Regression	2	2294.93	1147.46	73.96	0.00
Residual	47	729.15	15.51		
Total	49	3024.08			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	97.53	10.88	8.96	0.00	75.64	119.42
Price	-2.52	0.25	-10.24	0.00	-3.01	-2.02
Income	2.11	0.26	8.12	0.00	1.59	2.63

Table 3-5

The estimated demand equation is $Q = 97.53 - 2.52P + 2.11M$. This equation says that increasing *price* by $1 decreases quantity demanded by 2.52 units. Likewise, increasing *income* by $1,000 increases demand by 2.11 units. Since the *t*-statistics for each of the variables is greater than 2 in absolute value, *price* and *income* are significant factors in determining quantity demanded. The *R*-square is reasonably high, suggesting that the model explains 76 percent of the total variation in the demand for KBC microbrews.

MICHIGAN
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.63
R Square	0.40
Adjusted R Square	0.37
Standard Error	10.59
Observations	50

ANOVA

	degrees of freedom	*SS*	*MS*	*F*	*Significance F*
Regression	2	3474.75	1737.38	15.51	0.00
Residual	47	5266.23	112.05		
Total	49	8740.98			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	182.44	16.25	11.23	0.0000	149.75	215.12
Price	-1.02	0.31	-3.28	0.0020	-1.65	-0.40
Income	1.41	0.35	4.09	0.0002	0.72	2.11

Table 3-6

The estimated demand equation is $Q = 182.44 - 1.02P + 1.41M$. This equation says that increasing *price* by $1 decreases quantity demanded by 1.02 units. Likewise, increasing *income* by $1,000 increases demand by 1.41 units. Since the *t*-statistics associated with each of the variables is greater than 2 in absolute value, *price* and *income* are significant factors in determining quantity demanded. The *R*-square is relatively low, suggesting that the model explains about 40 percent of the total variation in the demand for KBC microbrews. The *F*-statistic is zero, suggesting that the overall fit of the regression to the data is highly significant.

MINNESOTA
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.64
R Square	0.41
Adjusted R Square	0.39
Standard Error	16.43
Observations	50

ANOVA

	degrees of freedom	*SS*	*MS*	*F*	*Significance F*
Regression	2	8994.34	4497.17	16.67	0.00
Residual	47	12680.48	269.80		
Total	49	21674.82			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	81.70	81.49	1.00	0.32	-82.23	245.62
Price	-0.12	2.52	-0.05	0.96	-5.19	4.94
Income	3.41	0.60	5.68	0.00	2.20	4.62

Table 3-7

The estimated demand equation is $Q = 81.70 - 0.12P + 3.41M$. This equation says that increasing *price* by $1 decreases quantity demanded by 0.12 units. Likewise, a $1,000 increase in consumer *income* increases demand by 3.41 units. Since the *t*-statistic associated with *income* is greater than 2 in absolute value, it is a significant factor in determining quantity demanded; however, price is not a statistically significant determinant of quantity demanded. The *R*-square is relatively low, suggesting that the model explains 41 percent of the total variation in the demand for KBC microbrews.

MISSOURI
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.88
R Square	0.78
Adjusted R Square	0.77
Standard Error	15.56
Observations	50

ANOVA

	degrees of freedom	*SS*	*MS*	*F*	*Significance F*
Regression	2	39634.90	19817.45	81.81	0.00
Residual	47	11385.02	242.23		
Total	49	51019.92			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	124.31	24.23	5.13	0.00	75.57	173.05
Price	-0.79	0.58	-1.36	0.18	-1.96	0.38
Income	7.45	0.59	12.73	0.00	6.27	8.63

Table 3-8

The estimated demand equation is $Q = 124.31 - 0.79P + 7.45M$. This equation says that increasing *price* by $1 decreases quantity demanded by 0.79 units. Likewise, a $1,000 increase in *income* increases demand by 7.45 units. The *t*-statistic associated with *price* is not greater than 2 in absolute value; suggesting that price does not statistically impact the quantity demanded. However, the estimated *income* coefficient is statistically different from zero. The *R*-square is reasonably high, suggesting that the model explains 78 percent of the total variation in the demand for KBC microbrews.

OHIO
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.99
R Square	0.98
Adjusted R Square	0.98
Standard Error	10.63
Observations	50

ANOVA

	degrees of freedom	*SS*	*MS*	*F*	*Significance F*
Regression	2	323988.26	161994.13	1434.86	0.00
Residual	47	5306.24	112.90		
Total	49	329294.50			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	111.06	23.04	4.82	0.0000	64.71	157.41
Price	-2.48	0.79	-3.12	0.0031	-4.07	-0.88
Income	7.03	0.13	52.96	0.0000	6.76	7.30

Table 3-9

The estimated demand equation is $Q = 111.06 - 2.48P + 7.03M$. This equation says that increasing *price* by $1 decreases quantity demanded by 2.48 units. Likewise, increasing *income* by $1,000 increases demand by 7.03 units. Since the *t*-statistics associated with each of the variables is greater than 2 in absolute value, *price* and *income* are significant factors in determining quantity demanded. The *R*-square is very high, suggesting that the model explains 98 percent of the total variation in the demand for KBC microbrews.

WISCONSIN
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.999
R Square	0.998
Adjusted R Square	0.998
Standard Error	4.79
Observations	50

ANOVA

	degrees of freedom	*SS*	*MS*	*F*	*Significance F*
Regression	2	614277.37	307138.68	13369.30	0.00
Residual	47	1079.75	22.97		
Total	49	615357.12			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	107.60	7.97	13.49	0.00	91.56	123.65
Price	-1.94	0.25	-7.59	0.00	-2.45	-1.42
Income	10.01	0.06	163.48	0.00	9.88	10.13

Table 3-10

The estimated demand equation is $Q = 107.60 - 1.94P + 10.01M$. This equation says that increasing *price* by $1 decreases quantity demanded by 1.94 units. Likewise, increasing *income* by $1,000 increases demand by 10.01 units. Since the *t*-statistics associated with *price* and *income* are greater than 2 in absolute value, *price* and *income* are both significant factors in determining quantity demanded. The *R*-square is very high, suggesting that the model explains 99.8 percent of the total variation in the demand for KBC microbrews.

18. Table 3-11 contains the output from the linear regression model. That model indicates that R^2 = .55, or that 55 percent of the variability in the quantity demanded is explained by price and advertising. In contrast, in Table 3-12 the R^2 for the log-linear model is .40, indicating that only 40 percent of the variability in the natural log of quantity is explained by variation in the natural log of price and the natural log of advertising. Therefore, the linear regression model appears to do a better job explaining variation in the dependent variable. This conclusion is further supported by comparing the adjusted R^2s and the *F*-statistics in the two models. In the linear regression model the adjusted R^2 is greater than in the log-linear model: .54 compared to .39, respectively. The *F*-statistic in the linear regression model is 58.61, which is larger than the *F*-statistic of 32.52 in the log-linear regression model. Taken together these three measures suggest that the linear regression model fits the data better than the log-linear model. Each of the three variables in the linear regression model is statistically significant; in absolute value the *t*-statistics are greater than two. In contrast, only two of the three variables are statistically significant in the log-linear model; the intercept is not statistically significant since the *t*-statistic is less than two in absolute value. At P = \$3.10 and A = \$100, milk consumption is 2.029 million gallons per week $\left(Q_{milk}^{d} = 6.52 - 1.61(3.10) + .005(100) = 2.029\right)$.

SUMMARY OUTPUT LINEAR REGRESSION MODEL

Regression Statistics	
Multiple R	0.74
R Square	0.55
Adjusted R Square	0.54
Standard Error	1.06
Observations	100.00

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2.00	132.51	66.26	58.61	2.05E-17
Residual	97.00	109.66	1.13		
Total	99.00	242.17			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	6.52	0.82	7.92	0.00	4.89	8.15
Price	-1.61	0.15	-10.66	0.00	-1.92	-1.31
Advertising	0.005	0.0016	2.96	0.00	0.00	0.01

Table 3-11

SUMMARY OUTPUT LOG-LINEAR REGRESSION MODEL

Regression Statistics	
Multiple R	0.63
R Square	0.40
Adjusted R Square	0.39
Standard Error	0.59
Observations	100.00

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2.00	22.40	11.20	32.52	1.55E-11
Residual	97.00	33.41	0.34		
Total	99.00	55.81			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-1.99	2.24	-0.89	0.38	-6.44	2.46
ln(Price)	-2.17	0.28	-7.86	0.00	-2.72	-1.62
ln(Advertising)	0.91	0.37	2.46	0.02	0.18	1.65

Table 3-12

19. Given the estimated demand function and the monthly subscriptions prices, demand is 172,000 subscribers $\left(Q_{sat}^{d}=152.5-0.9(50)+1.05(30)+1.10(30)\right)$. Thus, revenues are \$8.6 million, which are not sufficient to cover costs. Revenues are maximized when demand is unit elastic $\left(.9\left(\frac{P_{sat}}{217-.9P_{sat}}\right)=1\right)$: Solving yields $P_{sat}=\$120.56$. Thus, the maximum revenue News Corp. can earn is \$13,080,277.76 $\left(TR=P\times Q=120.56\times\left(217-.9\times 120.56\right)\times 1000\right)$. News Corp. cannot cover its costs in the current environment.