

Technical Questions and Problems

1. In Gelate, Pennsylvania, the market for compact discs has evolved as follows. There are two firms that each use a marquee to post the price they charge for compact discs. Each firm buys CDs from the same supplier at a cost of \$5.00 per disc. The inverse market demand in their area is given by $P = 10 - 2Q$, where Q is the total output produced by the two firms.
 - a. Solve for the Bertrand equilibrium price and market output.
 - b. Would your answer differ if the products were not perfect substitutes? Explain.

Answer:

- a. $P = MC = \$5$. To find industry output, we find Q such that $P = 5 = 10 - 2Q$. Solving for Q gives us industry output of 2.5 units.
 - b. When goods are perfect substitutes, firms are forced to charge a price equal to marginal cost, otherwise they sell nothing. However, if consumers view the goods as heterogeneous (differentiated products) a firm does not lose the entire market if it prices above another firm's price.
2. The (inverse) demand in a Cournot duopoly is $P = a - b(Q_1 + Q_2)$ and cost are $C_1(Q_1) = c_1Q_1$ and $C_2(Q_2) = c_2Q_2$. Show that the Cournot equilibrium levels of output are $Q_1 = \frac{a + c_2 - 2c_1}{3b}$ and $Q_2 = \frac{a + c_1 - 2c_2}{3b}$.

Answer:

Equating $MR = MC$ for firm one yields $a - b(Q_1 + Q_2) - bQ_1 = c_1$. This yields a reaction function for firm one of

$$Q_1 = \frac{a - bQ_2 - c_1}{2b}.$$

Similarly, equating $MR = MC$ for firm two yields $a - b(Q_1 + Q_2) - bQ_2 = c_2$, so two's reaction function is

$$Q_2 = \frac{a - bQ_1 - c_2}{2b}.$$

Solving these two equations simultaneously gives us the desired result.

3. The market for widgets consists of two firms that produce identical products. Competition in the market is such that each of the firms independently produces a quantity of output, and these quantities are then sold in the market at a price that is determined by the total amount produced by the two firms. Firm 2 is known to have a cost advantage over Firm 1. A recent study found that the (inverse) market demand curve faced by the two firms is $P = 280 - 2(Q_1 + Q_2)$ and costs are $C_1(Q_1) = 3Q_1$ and $C_2(Q_2) = 2Q_2$.
 - a. Determine the marginal revenue for each firm.
 - b. Determine the reaction function for each firm.
 - c. How much output will each firm produce in equilibrium?

d. What are the equilibrium profits for each firm?

Answer:

- a. $MR_1 = 280 - 2Q_2 - 4Q_1$ and $MR_2 = 280 - 2Q_1 - 4Q_2$.
- b. Firm one will produce such that $MR_1 = MC_1$, or $280 - 2Q_2 - 4Q_1 = 3$. Thus, the reaction function of firm one is $r_1(Q_2) = 69.25 - .5Q_2$. Firm two will produce such that $MR_2 = MC_2$, or $280 - 2Q_1 + 4Q_2 = 2$. The reaction function for firm two is thus $r_2(Q_1) = 69.5 - .5Q_1$.
- c. Solving Q_1 and Q_2 from the two reaction functions, we have

$$Q_1 = \frac{280 - 6 + 2}{6} = 46$$

$$Q_2 = \frac{280 - 4 + 3}{6} = 46.5$$

$$P = 280 - 2(Q_1 + Q_2) = 95.$$

- d. $\Pi_1 = 95(46) - 3(46) = \$4,232$; $\Pi_2 = 95(46.5) - 2(46.5) = \$4,324.5$.

4. When MCI announced a price discount plan designed to induce small firms to use its services, the price of its stock immediately declined. Why do you think the stock market reacted negatively to MCI's plan to attract new customers?

Answer:

The most likely reason the market did not respond favorably to MCI's plan is that investors recognized the market for long-distance services is oligopolistic; competitors like AT&T would likely react to MCI's plan by changing their own pricing structure. In fact, this is precisely what did happen; 6 days after the MCI announcement, AT&T followed with a similar plan. Effectively, MCI's action initiated a "price war" that parallels our analysis of Bertrand competition.

5. The inverse demand curve for a Stackelberg duopoly is $P = 10,000 - 6Q$. The leader's cost structure is $C_L(Q_L) = 15Q_L$. The follower's cost structure is $C_F(Q_F) = 25Q_F$.
- a. Determine the reaction function for the follower.
 - b. Determine the equilibrium output levels for both the leader and the follower.
 - c. What are the profits for the leader? For the follower?

Answer:

- a. The follower's reaction function is $Q_F = 831.25 - .5Q_L$.
- b. The leader's output is

$$Q_L = \frac{a + c_F - 2c_L}{2b} = \frac{10,000 + 25 - 30}{12} = 832.9.$$

Similarly, $Q_F = 831.25 - .5(832.9) = 414.8$ units. Thus, the price of output is given by $P = 10,000 - 6(832.9 + 414.8) = \$2,513.8$.

- c. $\Pi_L = 2513.8(832.9) - 15(832.9) = \$2,081,250.5$; $\Pi_F = \$1,032,354.2$.

6. What real-world evidence would lead you to believe that firms were acting as Cournot oligopolists? Stackelberg oligopolists? Bertrand oligopolists?

Answer:

Evidence of Cournot oligopoly would be a situation where firms make quantity-setting decisions. Evidence of Stackelberg behavior includes one firm setting output prior to other firms in the market, who take the leader's output as given. Evidence of Bertrand oligopoly would be severe price competition, with low prices and profits.

7. Zelda Industries is the only firm of its kind in the world. Due largely to historical accident, it began producing streganomas in 1985 in a vacant warehouse. Virtually anyone with a degree in college chemistry could easily replicate the firm's formula, which is not patent protected. Nonetheless, since 1985 Zelda has averaged accounting profits of 6 percent on investment. This rate is comparable to the average interest rate that large banks paid on deposits over the period. Do you think Zelda is earning monopoly profits? Why?

Answer:

No. In fact, Zelda could have invested funds over the period at 6 percent. Its accounting profits of 6 percent translate into zero economic profits. Most likely, Zelda operates in a contestable market, and is disciplined by the threat of entry by other firms. Therefore, Zelda cannot charge prices in excess of marginal cost.

8. You are the manager of a firm in a new industry. You have gotten the jump on the only other producer in the market. You know what your competitor's cost function is, and it knows yours. Your products, although different to experts, are indistinguishable to the average consumer. Your marketing research team has provided you with the following market demand curve: $Q = 1,250 - .5P$. Your cost function is $C_A(Q_A) = 8Q_A$. Your competitor's cost function is $C_B(Q_B) = 6Q_B$. Your diligent effort will allow you to decide how much of your product to provide and allow you to place it on the market shortly before your competitor will be able to make its product available for sale. What output level will you choose, and what price will you charge? Explain.

Answer:

You are clearly a Stackelberg leader, and should set the Stackelberg output to maximize your profits. To use the formulas in the text, you first need to convert the given demand equation ($Q = 1250 - .5P$) into the corresponding inverse demand function. To do this, solve the given demand equation for P to get $P = 2500 - 2Q$. Based on this inverse demand function, use the formula in the text to solve for your Stackelberg output as the leader. This is given by the formula $Q_A = (a + c_B - 2c_A)/2b = (2500 + 6 - 2(8))/4 = 622.5$ units. The follower will produce $Q_B = (a - c_B)/2b - .5Q_A = [2500 - 6]/4 - .5(622.5) = 312.25$ units. Thus, the price of output will be $P = 2500 - 2(622.5 + 312.25) = \630.50 .

9. You are a potential entrant into a market that previously has had entry blocked by the government. Your market research has estimated that the inverse market demand curve for this industry is $P = 22,500 - 75Q$, where $Q = \sum_{i=1}^n Q_i$. You estimate that if you enter the market, your own cost function will be $C_y(Q_y) = 3,000Q_y$. The government has invited your firm to enter the industry, but it will require you to pay a one-time license fee of \$100,000. You do not know the cost functions of the firms presently in the market; however, the price is now \$16,000. Last year 87 units were sold by existing firms. Would you choose to enter this market? Explain.

Answer:

In this case, your inverse demand function remains at $P = 22,500 - 75(87) - 75Q_y = 15,975 - 75Q_y$. Equating $MR = MC$ yields $15,975 - 150Q_y = 3,000$. Solving for Q_y yields the profit-maximizing output by the entrant, assuming existing firms hold output constant: $Q_y = 4.5$ units. The corresponding price is $P = 22,500 - 75(87 + 4.5) = \$15,637.5$. Profits if you enter (net of the license fee) are $\$15,637.5(4.5) - 3,000(4.5) - 100,000 = -\$98,481.25$. Thus, it would not pay to enter the industry if you expect rivals to maintain their present output.

10. Compare and contrast the output levels and profits for the Cournot, Stackelberg, and Bertrand models. Use the following cost and demand conditions for your comparison, and suppose there are two firms: $P = 1,500 - 10Q$. Each firm has a marginal cost of \$20 and fixed costs of zero.

Answer:

	Firm One's Output	Firm Two's Output	Firm One's Profit	Firm Two's Profit
Cournot	49.33	49.33	\$24,338	\$24,338
Stackelberg	(leader) 74	(follower) 37	\$27,380	\$13,690
Bertrand	74	74	\$0	\$0

11. Over the past 20 years, the 12 members of the Organization of Petroleum and Exporting Countries have made repeated attempts to restrict output in order to maintain high crude oil prices. Between 1990 and 1995, however, crude oil prices have dropped by about 20 percent, due in part to increased production from the former Soviet Union, Latin America, Asia, and the North Sea. In light of these increases in oil production from non-OPEC countries, what must OPEC do to maintain the price of oil at its desired level? Do you think this will be easy for OPEC to do? Explain.

Answer:

OPEC must decrease its total quantity of oil produced by the amount that non-OPEC countries increase their oil production. This will be difficult for OPEC to do. As

demand and cost conditions change, old collusive agreements no longer suit the new environment. Moreover, even in the absence of changing demand or cost conditions, each firm has an incentive to cheat on collusive agreements, as shown in the text.

12. In the late 1990s, Chrysler announced a new incentive program on its minivans that included subsidized interest rates and cash allowances. Under the plan, consumers could enjoy financing rates as low as 4.9 percent, as well as a \$500 cash allowance toward the lease or purchase of a new minivan. What changes in sales would you anticipate if you were the manager of a Dodge/Plymouth franchise? Why?

Answer:

Other things equal, your sales would rise. However, given the oligopolistic nature of the industry, you should anticipate that automakers like General Motors and Ford will likely counter with similar incentive programs, which will mitigate to some extent the sales increase you otherwise would have enjoyed.

13. Orion and Zeda are the only producers of a unique product that sold in a market where the inverse demand curve is $P = 200 - 2Q$. The firms produce identical products and have identical cost functions given by $C(Q_i) = 4Q_i$. The managers of each firm must decide on their outputs on Monday morning and then bring products to market by noon.
- What is each firm's marginal revenue? Marginal cost?
 - Equate each firm's marginal revenue to marginal cost.
 - Use your result in part (b) to solve for each firm's reaction function.
 - Use your results in part (c) to solve for the Cournot equilibrium levels of output for each firm.

Answer:

- $MR_O = 200 - 4q_o - 2q_z$; $MR_Z = 200 - 2q_o - 4q_z$; $MC_O = MC_Z = 4$.
- $MR_O = MC_O \rightarrow 200 - 4q_o - 2q_z = 4$; $MR_Z = MC_Z \rightarrow 200 - 2q_o - 4q_z = 4$.
- $q_o = r_o(q_z) = \frac{200 - 4}{4} - \frac{2}{4}q_z = 49 - .5q_z$ and
 $q_z = r_z(q_o) = \frac{200 - 4}{4} - \frac{2}{4}q_o = 49 - .5q_o$
- Substituting q_z into q_o from part (c) yields an equilibrium output for Orion of 32.7 units. Likewise, substituting q_o into q_z from part (c), equilibrium output for Zeda is 32.7 units.

14. You are the CEO of ClipIt, a paper clip manufacturer. Your company enjoys a patented technology that allows it to produce paper clips faster and at a lower cost than your only rival, FastenIt. ClipIt uses this advantage to be the first to choose its profit-maximizing output level in the market. The inverse demand function for paper clips is $P = 500 - 2Q$, ClipIt's costs are $C_C(Q_C) = 2Q_C$, and FastenIt's costs are $C_F(Q_F) = 4Q_F$.

- What is ClipIt's profit-maximizing output level? FastenIt's?

- b. What is the market's equilibrium price?
- c. How much profit does each firm earn?
- d. Ignoring antitrust considerations, would it be profitable for your firm to merge with FastenIt? If not, explain why not; if so, put together an offer that would permit you to profitably complete the merger.

Answer:

- a. Stackelberg leader's profit-maximizing quantity is 125 units; Stackelberg follower's profit maximizing quantity is 61.5 units.
- b. $P = \$127.00$.
- c. $\pi_C = \$15,625.00$; $\pi_F = \$7,564.50$.
- d. Assuming that the merged firms adopted ClipIt's patented technology that allows it to produce at a lower cost of $C_C(Q_C) = 2Q_C$, a merger would be profitable. Profits of the merged firm would be \$30,000.50 compared to joint (industry) profits of \$23,189.50 when firms compete in a Stackelberg setting.