LECTURE 24

MARKET DEMAND

QUESTIONS, ISSUES AND TOPICS TO BE COVERED:

– How do we aggregate individual demand to obtain the market demand?
  When can we do it?

– What are some useful metrics with which to describe demand curves and their responsiveness to price and income changes?

– What are some frequently used demand functions?
  What are their properties?

Skills to be Mastered:

- How to derive the market demand from individual demand curves.

- How to compute elasticities.

MARKET DEMAND CURVES

So far we have been dealing with the demand of a single individual.

Here, we derive the market demand from the individual demand curves.

Suppose there are 2 goods (Apples and Oranges) and 2 individuals.

Demand of individual 1: $A_1 = d_A^1(P_A, P_O, I_1)$

Demand of individual 2: $A_2 = d_A^2(P_A, P_O, I_2)$

Market Demand (for apples) is the sum of the individual demands.

$$A = A_1 + A_2 = d_A^1(P_A, P_O, I_1) + d_A^2(P_A, P_O, I_2)$$

or

$$A = D_A(P_A, P_O, I_1, I_2)$$

OBSERVATION 1: Both individuals are assumed to face the *same* prices.

Each person is assumed to be a price-taker, i.e., no haggling, no special discounts, no ability to influence price.

DISCUSSION: If there the above were not true, it would not be reasonable to talk about a single market for both individuals: There would a separate market for each one of them.

EXAMPLE: The government by purchasing of supplies in large quantities gets special prices. It is not reasonable to include it as part of the "market" demand for the type of products in which it can influence the price it pays.

OBSERVATION 2: In general, the demand for apples does not demand on aggregate consumer income.

It depends directly on each consumer's income.

This means that distribution of income matters.

<u>Example</u>: <u>Cobb-Douglas</u>.

Consider a good *X* and a market that consists of two individuals.

Let the individuals' preferences be

$$U_1 = X^{\alpha_1} Y^{1-\alpha_1} \qquad \text{and} \qquad U_2 = X^{\alpha_2} Y^{1-\alpha_2}$$

where α_i is the relative intensity of preference of consumer *i* for *X*. [Note that since the elasticity of scale has no meaning for utility functions, I can arbitrarily fix it to 1.]

Then, the demand for the good by each individual is given below

$$X_1 = \frac{I_1}{P_X} \alpha_1 \qquad \text{and} \qquad X_2 = \frac{I_2}{P_X} \alpha_2$$

where I_i is the income of person i.

The market demand is the sum of the individual demands:

$$X = X_1 + X_2 = \frac{I_1}{P_X} \alpha_1 + \frac{I_2}{P_X} \alpha_2$$

Factoring out the price of X we have:

$$X = \frac{1}{P_X}\left(\alpha_1 I_1 + \alpha_2 I_2\right)$$

OBSERVATION: We can not, in general, write the demand for X as a function of aggregate income $I = I_1 + I_2$.

Suppose: $\alpha_1 > \alpha_2$.

This means: Consumer 1 has a higher propensity to spend on good *X* out of his income than consumer 2 does.

Then: An extra dollar of income for consumer 1 will have a greater impact on the consumption of *X* than an extra dollar for consumer 2.

Example: A change in the tax code that results in a redistribution of money from consumer 1 to consumer 2, leaving total income unaffected would lead to a reduction in the total demand for X if $\alpha_1 > \alpha_2$.

QUESTION: When can we describe market demand as a function of total income ?

CASE 1: When marginal propensity to spend on X is the same for all individuals.

[$\alpha_1 = \alpha_2$]

CASE 2: When the distribution of income is not affected as income increases.

[$I_i^{new} = t\ I_i^{old}$]

Consider a proportional change of income to t times the original income.

Then:

$$X_{new} = \frac{1}{P_X}\left(\alpha_1 \; I_1^{new} + \alpha_2 \; I_2^{new}\right)$$

$$= \frac{1}{P_X}\left(\alpha_1 \; t \; I_1 + \alpha_2 \; t \; I_2\right)$$

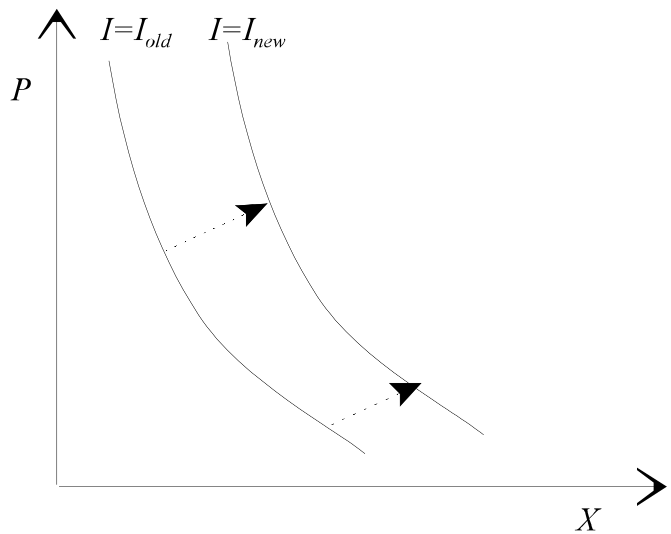
$$= t \; \frac{\alpha_1 \; I_1 + \alpha_2 \; I_2}{P_X}$$

$$= t \; X_{old}$$

When consumers have Cobb-Douglas preferences, an increase of income by a factor of t will increase consumption by a factor of t.

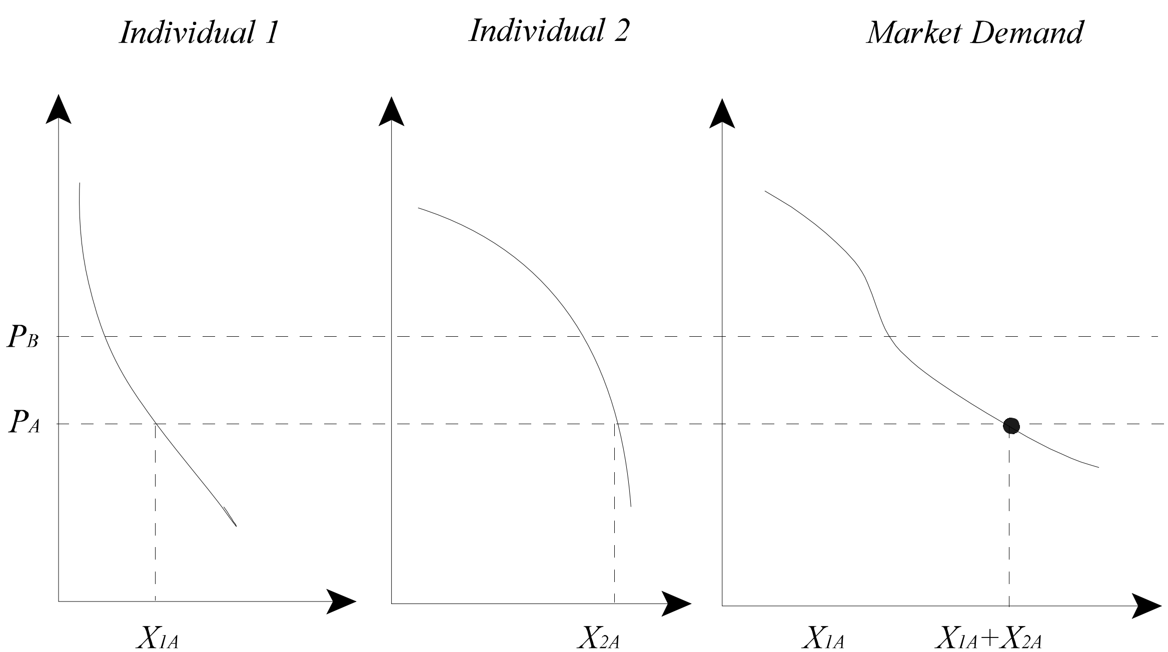
Recall the demand shifts you have learned in Principles.

The claim that market demand shifts out when income increases from I_{old} to I_{new} is valid (for normal goods) when *relative* income remains constant.



GEOMETRIC CONSTRUCTION OF MARKET DEMAND

Geometrically, the market demand is obtained by the *horizontal* addition of the individual demand curves.



NOTE: Individual and market demand functions are not only affected by changes in economic variables, but also by *changes in people's preferences*.

Example: The gradual shift from consumption of beef into consumption of chicken has more to do with changing consumer tastes than with changes of prices and income.

In the Cobb-Douglas utility example, a change in preferences would manifest itself as a change in the parameter α.

A Numerical Example.

Suppose individual demands for good X are given by

$$X_1 = 10 - 2\ P_X + 0.1\ I_1 + 0.5\ P_Y$$

and

$$X_2 = 17 - P_X + 0.5\ I_2 + 0.5\ P_Y$$

The market demand is the addition of these two demands:

$$X = 27 - 3\ P_X + 0.1\ I_1 + 0.5\ I_2 + P_Y$$

CAUTION: It only makes sense to add these two demand functions if each consumer is demanding a positive amount.

For some combinations of prices and income, the expressions for X_1 or X_2 will yield negative numbers.

In this case, actual demand is zero. You can not buy less than zero units of something. !

Adding the two demands in that case would be wrong, because we would be adding a negative number for a person's demand of someone, instead of adding zero.

Plotting this Example in a Graph.

Since we only plot a graph in two dimensions, we must fix some values for P_Y and I_i.

Suppose $I_1 = 40$, $I_2 = 2$, $P_Y = 4$.

The demand functions then become:

$$X_1 = 10 - 2\,P_X + 4 + 2$$
$$= 16 - 2\,P_X$$

and

$$X_2 = 17 - P_X + 1 + 2$$
$$= 20 - P_X$$

Since we normally plot the price on the vertical axis, it is best to first solve the individual demand equations for price.

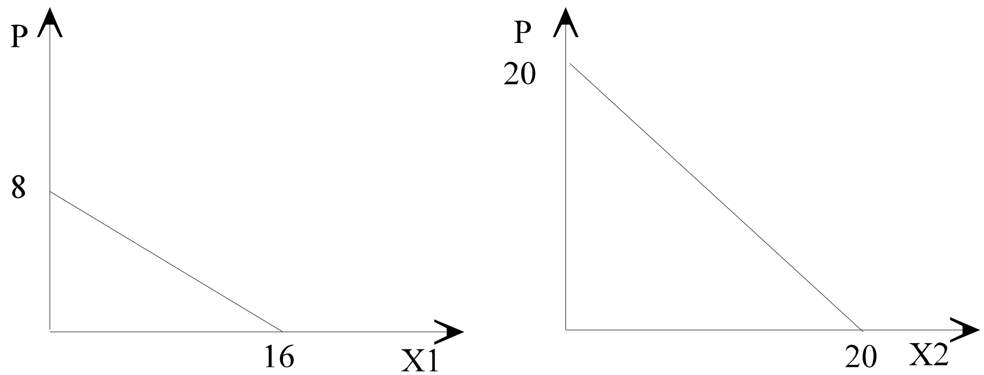
This yields:

$$P_X = 8 - \frac{1}{2} X_1$$

for individual 1, and

$$P_X = 20 - X_2$$

for individual 2.



NOTE: Individual 1 demands zero units of X when P_X reaches 8.

Individual 2 demands zero units of X when P_X reaches 20.

THEREFORE: Market demand, X, is equal to

$$X = \begin{cases} 0 & \text{if } \mathrm{P_X} > 20 \\ X_2 & \text{if } 20 > \mathrm{P_X} > 8 \\ X_1 + X_2 & \text{if } 8 > \mathrm{P_X} \end{cases}$$

The market demand is:

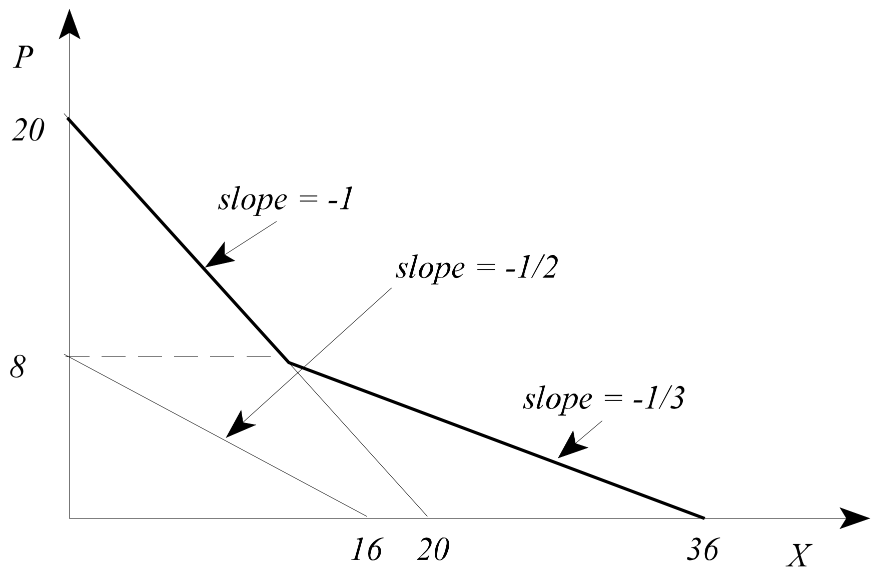
$$X = 0 \qquad \text{if } \mathrm{P_X} > 20$$

$$X = 20 - P_X \qquad \text{if } 20 > \mathrm{P_X} > 8$$

$$X = 36 - 3\,P_X \text{ if } 8 > \mathrm{P_X}$$

NOTE: Solving the last equation for P_X yields

$$P_X = 12 - \frac{1}{3} X$$



OBSERVATION: The slope of the demand curve is **not** the average of the slopes of the individuals demand curves.

THE CONCEPT OF DEMAND ELASTICITY

It is frequently useful to summarize how changes in an economic variable affect some other economic variable.

Example: Describe how an income change affects the demand for beef.

Idea: Look at the derivative of the demand for beef with respect to income.

For instance, it may be that an increase in income by 1 million dollars results in an increase in beef consumption by 500 kilos.

Problem: The numerical value of the response depends on the units of measurement.

If we were to measure the weight in tons, then the impact of income on the consumption of beef would be 0.5 tons.

This is not a huge problem if we limit ourselves to discussing responses to income changes of goods measured in units of weight.

It becomes a bigger problem if we want to compare responses of goods measured in different ways to a wide variety of economic variables.

EXAMPLE 1: We want to compare the response of beef, which is measured in units of weight, to an income change, with the response of automobile consumption to an income change.

Automobiles are measured in number of units sold, so there is no way, by re-scaling the units of measurement to make a comparison across the two markets.

EXAMPLE 2: We want to compare the responsiveness of beef to income changes to the responsiveness of beef to price changes.

Even though it is possible to use the same units of measurement for price and income, it would not be a reasonable way to compare responsiveness that way.

In order to be able to do these comparisons, we need a measure of response that is *independent* of units.

SOLUTION: Define the concept of elasticity.

Suppose an economic variable B is a function of a variable A and perhaps other variables as well:

$$B = f(A, \ldots).$$

Then the elasticity of B with respect to A is

$$e_{B,A} = \frac{\Delta \ \% \ in \ B}{\Delta \ \% \ in \ A} = \frac{\frac{\Delta \ B}{B}}{\frac{\Delta \ A}{A}} = \frac{\partial B}{\partial A} \frac{A}{B}$$

This measure is by construction dimensionless.

It does not depend on any units, since a percentage has no units.

(a) Price Elasticity of Demand.

The price elasticity of demand is:

$$e_{Q,P} = \frac{\Delta \% \ in \ Q}{\Delta \% \ in \ P} = \frac{\partial Q}{\partial P} \frac{P}{Q}$$

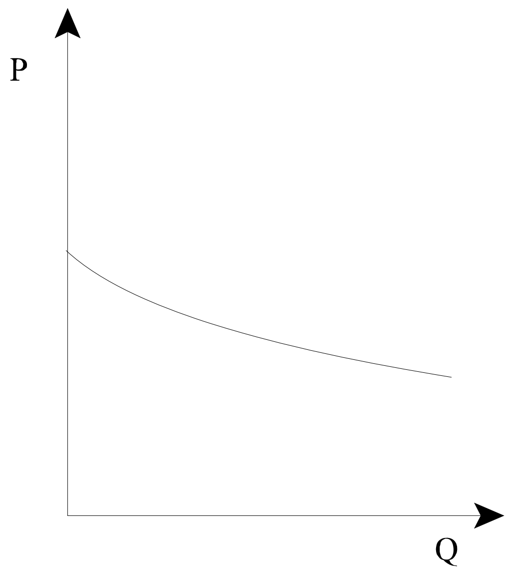
Classification of demand according to price elasticity:

$e_{Q,P} < -1$ elastic

$e_{Q,P} = -1$ unit elastic

$e_{Q,P} > -1$ inelastic

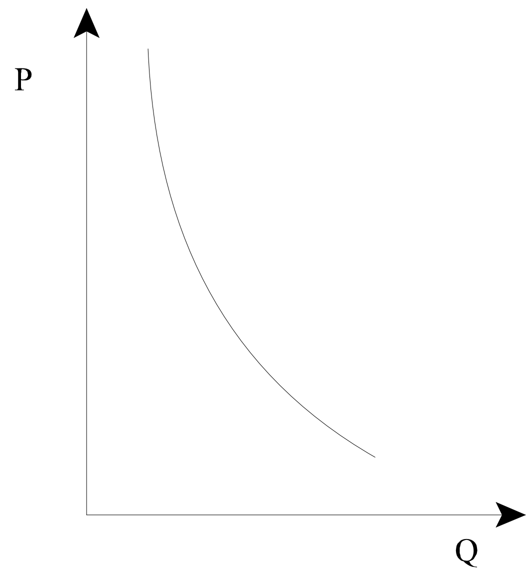
Elastic and inelastic demand look respectively like this



Price goes up by 1%

Q goes down by more than 1%

Industry revenues decrease.



Price goes up by 1%

Q goes down by less than 1%

Industry revenues increase.

NOTE: The price elasticity of demand is equal to the inverse of the elasticity price with respect to quantity:

$$e_{Q,P} = \frac{1}{e_{P,Q}}$$

Price Elasticity and Total Spending on a Good.

A. Consider a price increase for a good with unit demand elasticity:

The price increase leads to a decrease in consumption of equal proportions leaving total spending on the good unchanged.

B. Consider a price increase for a good with elastic demand:

The price increase leads to a (proportionately) larger decrease in consumption for the good, reducing total spending on that good.

C. Consider a price increase for a good with inelastic demand:

The price increase lead to a (proportionately) smaller decrease in consumption, increasing total spending on that good.

RECAP: Demands are defined as elastic or inelastic based on whether total spending on the good increases or decreases following a price increase.

(b) Income Elasticity of Demand.

The income elasticity of demand measures the sensitivity of market demand to income increases.

Given our previous discussions on the subject, these are to be interpreted as increases in total income that leave *relative* income (that is, the distribution of income) unchanged.

$$e_{Q,I} = \frac{\partial Q}{\partial I} \frac{I}{Q}$$

(c) Cross-Price Elasticities.

Cross-price elasticities measure the sensitivity of demand to a price change of another good.

For instance,

$$e_{Q_X, P_Y} = \frac{\partial Q_X}{\partial P_Y} \frac{P_Y}{Q_X}$$

measures the percent-change in the demand for *X* when the price of *Y* increases by one percent.

TYPES OF DEMAND CURVES

Two types of market demand curves are frequently used because of their simplicity.

(i) Linear Demand.

The equation of a linear demand is

$$P = \alpha - \beta Q$$

Properties: The slope of the linear demand is constant.

The price elasticity varies from (minus) infinity to zero.

Solving the demand equation for quantity yields:

$$Q = \frac{\alpha}{\beta} - \frac{1}{\beta} P$$

Then, the price elasticity is

$$e_{Q,P} = \frac{\partial Q}{\partial P}\frac{P}{Q}$$

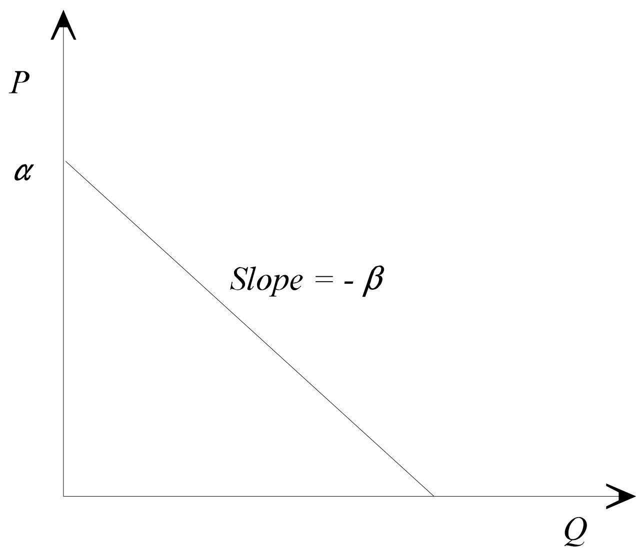
$$= -\frac{1}{\beta}\frac{P}{\frac{\alpha}{\beta} - \frac{1}{\beta}P}$$

$$= -\frac{P}{\alpha - P}$$

Clearly, when P=0 the elasticity is equal to zero.

As price increases, the elasticity decreases (the absolute value of the elasticity increases).

When price approaches α the elasticity approaches minus infinity.



(ii) Constant Elasticity of Demand Curve.

This demand is characterized by the equation

$$P = \alpha \, Q^{-\eta}$$

Properties: The slope of the demand varies from (minus) infinity to zero.

The price elasticity of demand is constant.

Observe that

$$e_{P,Q} = \frac{\partial P}{\partial Q} \frac{Q}{P}$$

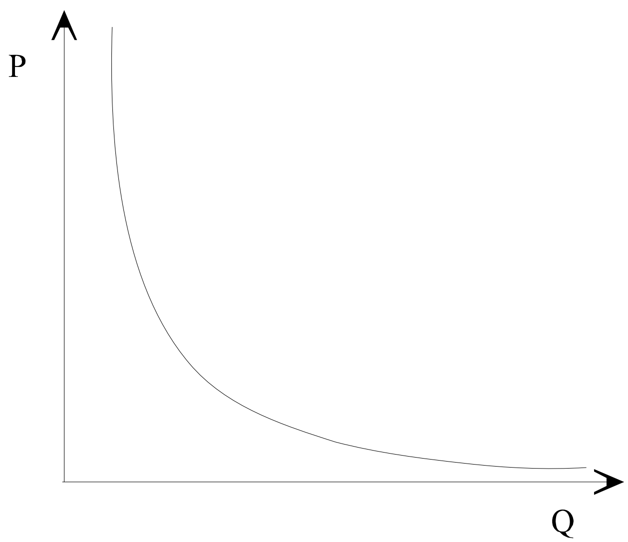
$$= -\alpha \eta Q^{-\eta-1} \frac{Q}{\alpha Q^{-\eta}}$$

$$= -\eta$$

Therefore, the price elasticity of demand is

$$e_{Q,P} = -\frac{1}{\eta}$$

A demand function with constant price elasticity looks like the one given in the figure below:



Observation: Suppose the demand curve is linear or constant elasticity *for only a portion* of its range.

Then, *for that portion*, one can still use the above formulas to compute the demand elasticity.

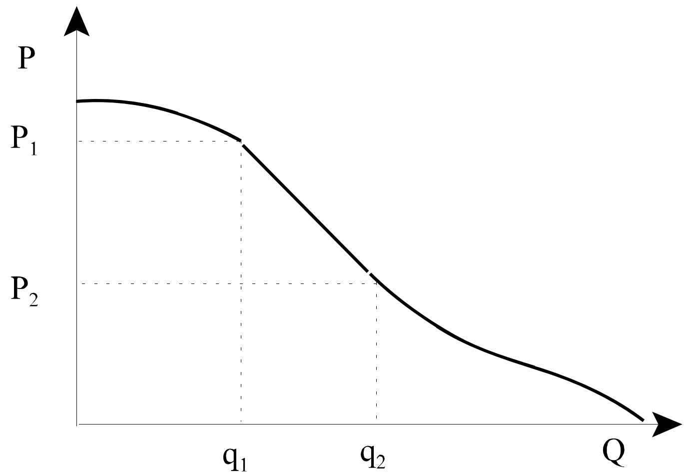
Why it works: Because the formulas use only slope information and the value of P and Q at a point, not information about the entire shape of the demand function.

Example. Suppose

$$P(q) = f(q) \qquad \text{for} \quad q < q_1$$

$$P(q) = \alpha - \beta q \qquad \text{for } q_1 < q < q_2$$

$$P(q) = g(q) \qquad \text{for } q_2 < q$$



Then

$$e_{Q,P} = -\frac{P}{\alpha - P} \qquad \text{for} \quad P_2 < P < P_1$$